Fractal clustering of metamorphic veins

Craig E. Manning
Department of Earth and Space Sciences,
University of California, Los Angeles, California 90024-1567

ABSTRACT

Metamorphic veins record the fracture-controlled flow of fluids throughout the oceanic and continental crust. I show that the spatial distributions of veins from three diverse metamorphic settings are fractal and self-similar. Vein densities were measured by counting the number of macroscopic veins intersected along linear transects. The localities included wollastonite-quartz veins in marbles, for which the fractal dimension D is 0.46, actinolite-chlorite veins in hydrothermally altered oceanic diabases (D = 0.81), and epidote-quartz veins in contact-metamorphosed basalts (D = 0.25–0.63). The fractal clustering of veins provides a geometric framework for understanding spatial and temporal patterns of fluid flow and mineral reaction during metamorphism.

INTRODUCTION

By transporting mass and helping to control the rates and extents of mineral-fluid reactions, H_2O-CO_2-NaCl fluids play an important role in metamorphism. The extent to which the fluid phase participates in metamorphism hinges in part on its spatial distribution in the rock matrix. Metamorphic fluids migrate through fractures that range from microscopic arrays with widths, lengths, and spacings similar in scale to constituent mineral grains, to macroscopic arrays with widths, lengths, and spacings on the order of tens of centimetres to metres. In a typical metamorphic event, the scale and abundance of fractures changes in time and space as a complex response to changes in pressure and temperature (Norris and Henley, 1976; Norton and Knight, 1977; Yardley, 1986; Manning and Bird, 1991). Throughout the event, the fractures are filled by minerals precipitated from flowing fluids. Key elements of the history of fluid flow are thus recorded for each metamorphic event in a crosscutting sequence of mineral-filled fractures, or veins, and associated reaction zones in the wall rock. The spacing of fractures controls the degree to which fluids pervade the rock matrix or are segregated into discrete fracture-controlled flow zones (e.g., Walther and Wood, 1984). However, there has been little effort toward developing a rigorous geometric framework within which to evaluate the spatial patterns of fractures and veins in the context of metamorphism.

Most fracture arrays in natural materials are not evenly spaced. Although fracture distributions may appear purely random, it has long been known that they cluster in groups or swarms to varying degrees. Recently, many geologic fracture sets have been shown to have fractal distributions (e.g., Barton and Larsen, 1985; Chilès, 1988; La Pointe, 1988; Barton and Hsieh, 1989; Velde et al., 1990, 1991). Other aspects of fractures are also fractal: experimental studies show that the spacings and surface roughness of laboratory-induced microfractures are fractal (Brown and Scholz, 1985; Hirata et al., 1987); fractal geometries characterize fault lengths and epicenter distributions in zones of active seismicity (e.g., Kagan and Knopoff, 1980; Aviles et al., 1987; Okubo and Aki, 1987); and the erosional patterns that develop in fractured rocks result in fractal topography (Norton and Sorenson, 1989). Below I show that veins from three diverse metamorphic environments also have fractal spacings. This implies that fracture-controlled metamorphic fluid flow has spatial structure over a range of scales. Recognition of this structure aids petrologic and geochemical studies of fluid-rock interaction.

FRACTAL ANALYSIS

Methods

A fractal set has a Hausdorff-Besicovitch, or fractal, dimension D greater than the topologic dimension of the objects that constitute the set (Mandlebrot, 1983; Turcotte, 1992). Fractures and veins exposed in an outcrop surface can be taken as lines with topologic dimension 1 in a plane with topologic dimension 2. If this plane were completely filled by fractures, D would be 2; so fractal veins or fractures on a two-dimensional surface are characterized by $1 < D < 2$ (Barton and Larsen, 1985; Chilès, 1988; La Pointe, 1988). However, characterization of vein densities is usually secondary to other mapping or sampling objectives; outcrops are not necessarily well exposed in all directions, a drill core being an extreme example, and generating two-dimensional vein maps is time intensive. For these reasons, vein data are frequently collected by using suitably oriented linear transects. The data in this investigation were similarly acquired. Intersections of veins with a transect define points on a line and are fractal if $0 < D < 1$.

The fractal dimension is obtained for linear vein transects by mapping the positions at which veins intersect transect lines. The map is then broken into equally spaced divisions of length r and the number of divisions N_r that contain one or more veins is recorded. After repetition for a range in r, D is related to r and N_r by

$$D = \frac{d(\log N_r)}{d(\log 1/r)}.$$

If D is a constant between 0 and 1 for a range in r, the veins compose a homogeneous fractal set and are self-similar or scale invariant over the included length scales. The magnitude of D is a measure of the manner in which the transect is filled by points: D increases with increasing degree of space filling and decreasing clustering of the veins.

Fractal analyses were conducted on veins from metabasalt and marble outcrops and from hydrothermally altered diabases of Ocean Drilling Program Hole 504B. In the outcrop vein surveys, a tape measure was laid out normal to the vein strike. The locations of intersections of each vein were then recorded, along with their widths and the minerals filling them. The characterization of vein densities in drill core is analogous in that the axis of measurement is a line, but here it is the bore hole. For each set, D was determined by linear least-squares analysis. Correlation coefficients R reflect the set's linearity, or self-similarity, not the uncertainty in D.

The range in r that can be investigated is an intrinsic property of the data sets and is defined by upper and lower cutoffs (Chilès, 1988). The upper cutoff is the value of r for which each segment is intersected by at least one vein. The lower cutoff is defined by the widths of the veins or the precision of the measurements of vein locations. Transect orientation can bias the inferred spatial structure of clustering (Terzaghi, 1965; La Pointe and Hudson, 1985; Velde et al., 1990). To avoid this bias, the outcrops were evaluated carefully to ensure that the veins defined regularly oriented sets for which linear transects would be appropriate and that no veins were subparallel to the tape measure. For veins sampled by drill core, no such guarantees exist. The orientation of the hole is usually unknown, and the extent to which the axis of measurement may parallel vein attitudes is undetermined. However, the sampling biases inherent in vein and fracture orientations in vertical bore holes (Newmark et al., 1985) can be ignored if vein orientations are random, which is the case for Hole 504B (Dick et al., 1992).

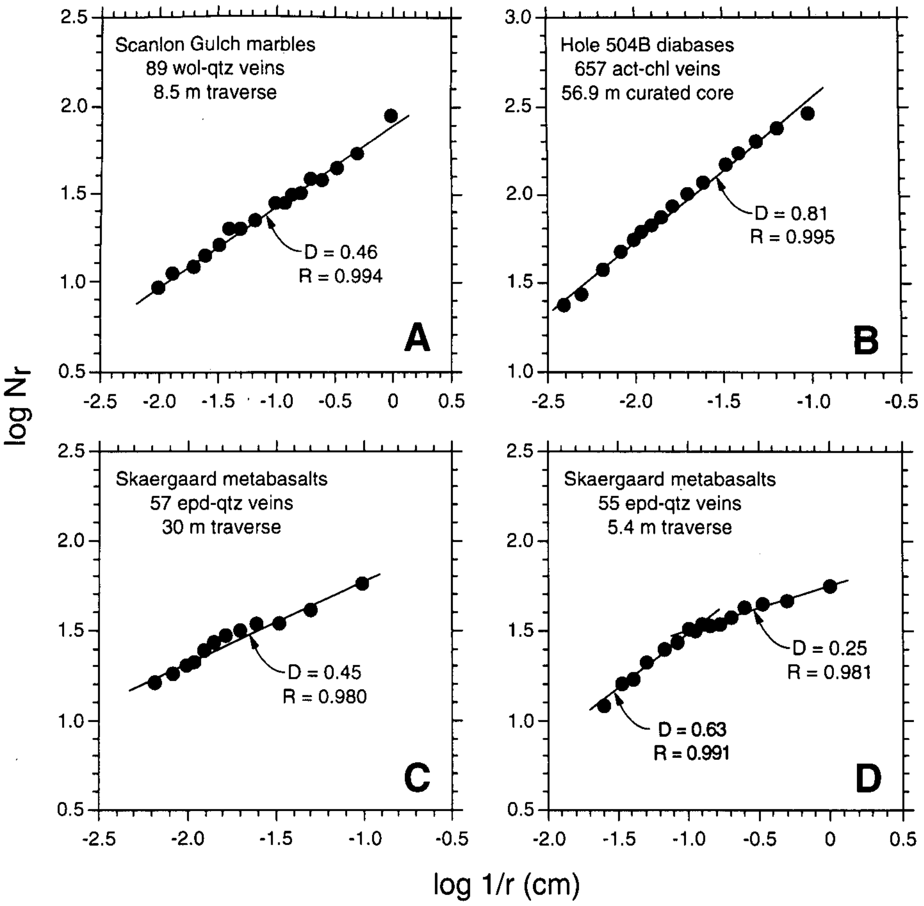

Figure 1. Variation in N_r with $1/r$ for four vein transects. Wol—wollastonite; qtz—quartz; act—actinolite; chl—chlorite; epd—epidote.

Wollastonite-Quartz Veins in Scanlon Gulch Marbles

Upper Paleozoic calc-silicates and marbles in the Scanlon Gulch area of the Old Woman Mountains, southeastern California, were thermally metamorphosed in a screen ~1 km thick between ~73 Ma granitic intrusions (Foster et al., 1989, 1992; Rothstein, 1990). Both lithologies contain abundant wollastonite, implying moderate to large time-integrated fluid fluxes and silica metasomatism during metamorphism. Subvertical veins trending 320–350 crosscut the calc-silicates and marbles. The veins postdate the peak in metamorphism and are filled by wollastonite, quartz, or both. An 8.5 m survey line oriented 065 in wollastonite + calcite marbles of the Supai Formation intersected 89 veins yielding an average vein density of 10.5/m. The wollastonite-quartz veins are <1 to 8 mm thick.

Figure 1A shows that $\log N_r$ is linear in $\log 1/r$ with $D = 0.46$ and $R = 0.994$. This means that the veins are self-similar over two orders of magnitude of r between 0.01 and 1 m. The lower cutoff of 1 cm was defined by the maximum vein widths of 8 mm. Transects of greater length are required to determine if self-similarity continues to higher r.

Actinolite-Chlorite Veins in Hole 504B Diabases

Sheeted dikes from ~1600 to 2000 m below the sea floor were recovered from Hole 504B during Leg 140 of the Ocean Drilling Program. Dikes variably altered to greenschist facies mineral assemblages are crosscut by randomly oriented hydrothermal veins (Dick et al., 1992). As noted by Dick et al. (1992), >98% of the veins in the core contain actinolite, chlorite, or both, and ≤5% albite, epidote, quartz, prehnite, titanite, and sulfides. These veins, referred to as actinolite-chlorite veins, are crosscut by rare epidote-quartz veins with trace sulfides. Because they formed later, the epidote-quartz veins were omitted from the analysis. The average density of actinolite-chlorite veins is 11.5/m.

During description of the veins in the drill core, the number of veins in each piece of core was noted rather than the exact locations of the veins relative to a reference line (Dick et al., 1992). This limits the minimum value of r to the maximum length of recovered pieces, which was ~10 cm. Figure 1B shows that $\log N_r$ varies linearly with $\log 1/r$ for veins in the 56.9 m of diabases recovered from the ~400 m interval drilled during Leg 140, giving $D = 0.81$ with $R = 0.995$. This linear variation suggests self-similarity over one order of magnitude of r.

Core samples reflect individual pieces from unknown but sequential positions within drilled intervals of known length. Errors are introduced into the fractal dimension of vein distribution because measurements include ~1 cm spacers added between pieces during curation. The added space makes the apparent D larger than the actual D, but this effect is small for such large data sets (La Pointe, 1988) and does not influence the precision of the self similarity.

Epidote-Quartz Veins in Skaergaard Metabasalts

Tertiary basalts in central East Greenland were contact metamorphosed by the Skaergaard intrusion (Bird et al., 1986; Manning and Bird, 1991; Manning et al., 1993). Veins filled by epidote and quartz are common in the aureole. Two traverses were conducted 1.5 and 2.0 km east of the intrusion where contact metamorphism led to greenschist facies assemblages. In both localities, the veins are 0.1–2 mm wide and vertical, and have a narrow range of strike (070–090). Figure 1C shows results for 57 veins intersected along a 30 m line striking 000 (1.9 veins/m). The strong linear correlation of $\log N_r$ in $\log 1/r$ over one order of magnitude of r (Fig. 1C) shows that the vein distribution is statistically self-similar with a fractal dimension of 0.45 ($R = 0.980$).

A second traverse 5.4 m long and oriented 355 intersected 55 epidote-quartz veins for an average vein density of 10.1/m. Figure 1D shows that the veins are fractal ($0 < D < 1$), but there are two similarity dimensions: at low r, $D = 0.25$ ($R = 0.981$); at high r, $D = 0.63$ ($R = 0.991$). The two values of D mean that the vein distributions scale with different fractal dimensions over different ranges in r. This could be a consequence of the two inferred fracturing episodes with the same mineral filling assemblage and orientation in this part of the Skaergaard's contact aureole (Manning and Bird, 1991).

FRACTAL CLUSTERING OF METAMORPHIC VEINS

The mapped vein arrays in Figure 1, A–C, are self-similar fractals over 1.5 to 2.5 orders of magnitude in r. Their self-similarity means that the spatial structure is scale invariant between the upper and lower cutoffs. The veins in Figure 1D display two distinct similarity dimensions. These fluid-flow channels thus have two levels of spatial structure, each of which is scale invariant within a distinct range in r.

For a given transect length, and number of veins, the fractal dimension increases with decreasing degree of clustering (La Pointe, 1988). Of the vein sets investigated, the outcrop surveys have lower D. These vein sets display more clustering and larger unveined regions. By contrast, the actinolite-chlorite veins in Hole 504B yield the largest observed D (0.81). The high D indicates a more pervasive set with less space between veins.

The fractal clustering of metamorphic veins can be appreciated by comparing the observed distribution of veins at Scanlon Gulch to the same number of purely random veins and evenly spaced veins (Fig. 2). Random veins differ from evenly spaced veins in that random, uneven spacings result in minor clusters. However, the natural pattern is substantially more clustered than the purely random pattern, presumably because initial zones of weakness persist throughout fracturing events. The variation in log N_r with log $1/r$ for evenly spaced veins displays two linear segments (Fig. 3). Where log $1/r$ is between -1 and 0, log N_r is constant and $D = 0$, the topological dimension of the veins; where log $1/r$ is between -2 and -1, log N_r decreases linearly such that $D = 1$, the topologic dimension of the transect. Evenly spaced veins therefore do not define a fractal set. Also shown in Figure 3 is the variation in N_r with $1/r$ for 89 veins distributed randomly over 8.5 m. Near the upper and lower cutoffs, D approaches limits of 1 and 0, respectively. Between these limits, $0 < D < 1$ and D varies continuously with r. Thus, random distributions are also not self-similar. A random distribution would imply that fractures were completely independent of one another. The difference between the synthetic random clustering and the self-similar clustering of the observed distribution emphasizes that natural fluid-flow channels are interdependent.

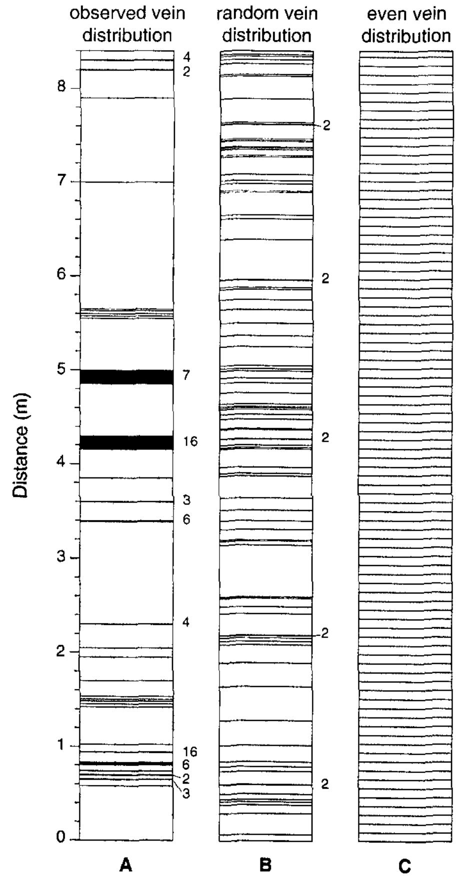

Figure 2. Comparison of observed distribution of 89 wollastonite-quartz veins along 8.5 m transect at Scanlon Gulch (A) with random (B) and even (C) distribution of same number of veins. Numbers to right of A and B are number of veins where they are too tightly clustered to be resolved. Shading around each vein represents hypothetical 5 cm halo of wollastonite (see text).

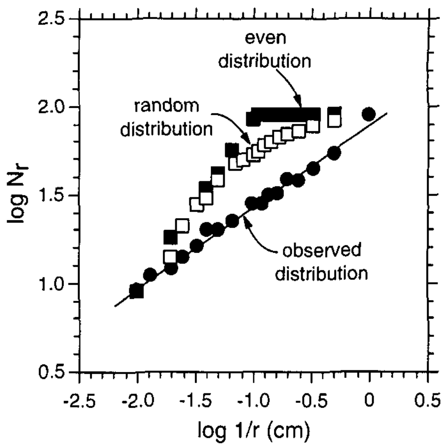

Figure 3. Variation in N_r with $1/r$ for vein distributions in Figure 2.

IMPLICATIONS FOR METAMORPHISM

The dominant sites of metamorphic fluid flow are fractures at some scale (e.g., Brenan, 1991). At microscopic scales, these fractures are represented by grain boundaries and intragranular cracks commonly recorded as healed fluid-inclusion arrays. At larger scales, flow channels are represented by macroscopic veins. Independent of the scale of observation, metamorphic veins record the time-integrated history of flow for all or part of a metamorphic event. That observed macroscopic vein sets are fractal provides a framework for understanding the spatial and temporal patterns of metamorphic fluid flow in the study areas.

Conceptual models for the evolution of fluid flow during metamorphic events suggest that fluid is pervasively distributed in microscopic flow channels in the rock matrix during prograde metamorphism. This permits the extensive mineral-fluid reaction that characterizes this part of the event. By contrast, localized mineral-fluid reaction during later retrograde metamorphism suggests that fluids become channeled into larger macroscopic fractures as the rocks cool. Treating metamorphic flow channels as fractal clusters is a better framework than the end-member view of channeled vs. pervasive flow in that it provides a quantitative measure for comparing and contrasting fluid flow in different areas and at different scales. Comparing D for crosscutting veins allows quantification of the transition from pervasive flow (high D) to more channeled flow (low D).

The recognition of fractal vein clustering also aids in the interpretation of the spatial patterns of the progress of metamorphic reactions. For example, if a water-rich fluid saturated with quartz were flowing in fractured calcite marbles at amphibolite facies conditions, the chemical potential gradient in the thermodynamic component SiO_2 would lead to diffusive haloes of wollastonite about each vein. This is a useful simplified model for fracture-controlled metamorphism at Scanlon Gulch. Assume in this illustration that fluxes were sufficient to cause 5 cm wollastonite haloes around each fracture (Fig. 2, stippled pattern) via the reaction $CaCO_3 + SiO_{2(aq)} \rightarrow CaSiO_3 + CO_2$. A simple measure of the extent of transformation of the outcrop to wollastonite is the reaction progress variable, ζ, which can be calculated from the number of moles of wollastonite divided by its stoichiometric reaction coefficient in the model reaction (e.g., Ferry, 1986). Assuming negligable Ca solubility, there will be a mole-for-mole transformation of calcite to wollastonite in the hypothetical fractured outcrop. The value of ζ will thus correspond to the area of the outcrop that is now wollastonite, and the pattern of metamorphism will vary with fracture distribution. Evenly spaced veins will be 10 cm apart, so symmetric 5 cm haloes about each vein in the hypothetical example will result in $\zeta = 1$ (Fig. 2A). Clustering will not affect the flux but will decrease the chemical potential gradients because of overlapping haloes, so $\zeta < 1$ for uneven distributions of the same number of veins. For purely random veins (Fig. 2B), ζ would be 0.65; for the observed, fractal distribution, ζ would be 0.29. The greater the clustering, the lower the fractal dimension of the fractures, the lower the value of ζ, and the greater the degree to which fluids migrate through the fractures without exchanging SiO_2 with the wall rocks.

Similar analyses may help to interpret spatial variations in isotopic exchange during metamorphism. For example, in an outcrop of metamorphic rock with $^{18}O/^{16}O$ modified by prograde fluid flow, variations in $\delta^{18}O$ between samples may be several per thousand. Although these variations might be attributed to random heterogeneities or selective retrogression, they may instead reflect fractal clustering of the flow channels through which fluids migrated.

Metamorphic veins reflect a time-integrated record of fluid flow. At any given time during the metamorphic event, only a subset of the observed vein array may have contributed to the permeability. Kagan (1991) showed that *D* for the distribution of earthquake epicenters increases asymptotically to a maximum value with increasing observation time. If this serves as a model for the time-dependent changes in *D* for metamorphic flow channels, it suggests that observed values of *D* are the maximum values that characterized a vein set. This implies that as a metamorphic event proceeds, successive fractures become less clustered, permitting more extensive fluid-rock interaction. In combination with insights into the distribution of fluid-rock reactions, these observations show that recognition of the fractal clustering of veins and fractures from diverse environments provides a new framework for investigating the spatial and temporal patterns of crustal fluid flow during metamorphism.

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