Planetary Surfaces

- Most surfaces are inaccessible and can be probed only with photons.
- Most surfaces are porous aggregates.
- Is the case about photons interacting with powders - a very tough subject.

- Photons can be scattered or absorbed. The absorbed ones heat the surface and lead to emission of longer λ photons.

![Graph with wavelengths and flux density](image)
Temperature - Heat Transport

Heat energy

\[ H = m \cdot c_p \cdot T \] [J]

\[ \text{Heat loss by conduction} \]

\[ \frac{dH}{dt} = r^2 \cdot k \cdot \frac{dT}{dr} \] [W/s]

Center to edge temp gradient

Cooling Time

\[ T \sim \frac{H}{H_0} \] [s]

\[ \sim \frac{\rho \cdot r^3 \cdot c_p \cdot T}{r^2 \cdot k \cdot (dT/dr)} \]

What to do about \( dT/dr \)?

Simplest approx is to set edge \( T = 0 \), \( T = T_{\text{middle}} \)

Then \( \frac{dT}{dr} \sim \frac{T}{r} \)
So \[ \tau \sim \frac{\rho r^3 c_p T}{r^2 k (T/r)} \]

\[ \sim \left( \frac{\rho c_p}{k} \right) \frac{r^2}{k} \]

\[ \tau \sim \frac{r^2}{k} \]

where \( k \left( \frac{k}{\rho c_p} \right) = \text{thermal diffusivity} \)

\[ [k] = \frac{J/s/m/k}{kg/m^3 K} = m^2/s = m^2s^{-1} \text{ (area/time)} \]

**eg:** Pea \( r \sim 3\) mm, \( k_{\text{pea}} \sim \frac{1}{10^3 10^3} \) \( = 10^{-6} \) m s^{-1}

\[ T_{\text{pea}} \sim \frac{(3 \times 10^{-3})^2}{10^{-6}} \sim 10 \text{ sec} \]

Potato; \( r \sim 3\) cm

\[ T_{\text{potato}} \sim 1000 \text{ sec} \]

in agreement wi. experience
\[ T \sim \frac{r^2}{K} \]

Distance travelled by heat through conduction in time \( T \) is
\[ r \sim \sqrt{K T} \]

The thermal diffusivity of dielectric solids of planetary interest is \( K \approx 10^{-6}\text{ m}^2\text{ s}^{-1} \). Metals are \( \approx 100 \times \) larger. Powders are smaller, by factor strongly dependent on porosity. (eg \( K \approx 10^{-7} \) or less).

eg: \( T = 1 \text{ day}, \quad K = 10^{-6}\text{ m}^2\text{ s}^{-1} \) (rocky planet)

\[ r \sim \sqrt{10^{-6} \times 10^5} \sim \sqrt{10^{-1}} = 0.3\text{ m} \]

Sun drives a T wave with scale 0.3 m due to \( \Theta \) rotation

eg: hot sand under a cold night-time beach
Heat conduction equation (1D)

\[
\frac{d^2 T}{dt} = \left( \frac{k}{\rho c_p} \right) \frac{d^2 T}{dr^2}
\]

can be solved for a planet surface illuminated by sun & rotating. Behavior depends on \( k \), not \( k, \rho, c_p \) separately.

Planetary scientists often use "thermal inertia",

\[
I = \sqrt{kgc_p}
\]

\[\text{[J} \text{m}^{-2} \text{K}^{-1} \text{kgm}^{-3} \text{J kg}^{-1} \text{K}^{-1} \text{]}^{1/2}
\]

\[= kg \text{m}^{-2} \text{K}^{-1} \text{kgm}^{-3} \text{J kg}^{-1} \text{K}^{-1} \]

\[= kg \text{ m}^{-1.5} \text{ K}^{-1.5} \text{ very ugly}
\]

\[I = \sqrt{kg^2c_p^2} = \sqrt{k^2c_p^2}
\]

eg. \( I \sim 1.10^{3} \text{ m}^{-1} \sim 10^{3} \text{ MKS units}

Fluffy regolith may have \( I < 100 \text{ MKS}

Thermal inertia is not a happy thing - physically diffusivity is what matters.
Solid rock (Paul Warren - 2011)

$\kappa (W/m \cdot K)$

$10^{-3}$

porosity $\% \rightarrow 70$

$T$ vs time on surface

peak $T \neq$ noon

$T$ vs depth

$\sqrt{K \tau} = l$

damped wave
period 1 day
amplitude $\exp(-d/l)$
Diurnal Effects

Skin depth
\[ l \sim \sqrt{KP} \]

Skin heat energy
\[ E = (4\pi r^2 p l) \ \cancel{C_p} \ T \]

\[ \text{mass of shell} \]

Heat is radiated to space at rate
\[ \dot{E} = 4\pi r^2 \varepsilon \sigma T^4 \]
(Black body)
\[ \varepsilon \text{ emissivity} \ (0 < \varepsilon < 1) \]

\[ \sigma = 5.67 \times 10^8 \text{ W m}^{-2} \text{ K}^{-4} \]

If rotation is fast, body will not have time to cool at night.

ie: \text{isothermal if} \ E \gg \dot{E} P

\[ \frac{4\pi r^2 p \sqrt{KP}}{C_p} \ T \gg 4\pi r^2 \varepsilon \sigma T^4 P \]

Square
\[ r^2 K \sqrt{P} C_p^2 \gg \varepsilon^2 \sigma^2 T^6 P \]

or
\[ P \ll \frac{P^2 K C_p^2}{\varepsilon^2 \sigma^2 T^6} \]
or \[ P \ll \frac{\rho k c_p}{\varepsilon^2 \sigma^2 T^6} \]

Substitute

\[ P \ll \frac{10^3 \cdot 0.1 \cdot 1000}{1.2 (5.7 \times 10^8)^2 (T/100)^6} \]

\[ P \ll \frac{8600 \text{ hr}}{(T/100)^6} \]

eg: 
- \( T = 100 \text{ K} \) (\( R \sim 10 \text{ AU} \), Saturn) \( P \ll 8600 \text{ hr} \)
- \( T = 300 \text{ K} \) (\( R \sim 1 \text{ AU} \), Earth) \( P \ll 10 \text{ hr} \)

So, asteroids experience significant diurnal \( T \) variation (so does Moon), but KBOs do not.

\[ T \uparrow \]

\[ R = 1 \text{ AU} \]

\[ R = 30 \text{ AU} \]

for exactly the same object properties.
Graphical representation of the thermal method for determining asteroid size and albedo.
Temperature

(a) Black Body in Radiative Equilibrium

\[
F_0 \frac{T_4^4}{4\pi R^2} = 4\pi \sigma G T^4
\]

Substituting \( F_0 = \frac{L_0}{4\pi R^2} \left[ \frac{W}{m^2} \right] \)

\[
T_{BB} = \left( \frac{L_0}{16\pi R^2 \sigma} \right)^{\frac{1}{4}} = \left( \frac{4 \times 10^{26}}{16\pi \times 5.67 \times 10^{-8}} \right)^{\frac{1}{4}} \frac{1}{\sqrt{R}}
\]

Express \( \Delta \) in AU, 1 AU = 1.5 x 10^11 m, then

\[
T_{BB} = \frac{278}{R_{AU}^{1/2}} [K]
\]

Example: \( R_{AU} = 1 \), \( T_{BB} = 278 K \); \( R_{AU} = 10 \) AU, \( T_{BB} \sim 88 K \)

\[ T_{BB} \propto \frac{1}{\sqrt{R}} \]
b) Spherical Planet w/ Finite Albedo & Emissivity, but Isothermal

\[ F_0 (1-A) \pi a^2 = 4\pi a^2 \epsilon \sigma T^4 \]

\[ T = \left( \frac{(1-A)}{\epsilon} \right)^{\frac{1}{4}} T_{BB} \]

Bond Albedo

\( A = \text{fraction of incident power that is absorbed} \)

\( 0 \leq \epsilon \leq 1 \)

\( T_{BB} \)

\[ F_0 (1-A) \pi a^2 = 2\pi a^2 \sigma \epsilon T^4 \]

radiating area

\( 2 \times \) absorbing

\( T = 0 \) permanent night

\( T = \text{permanent day} \)

c) Spherical planet, non-rotating, A, \( \epsilon \), no atmosphere

d) Flat plate planet

\[ F_0 (1-A) \pi a^2 = \pi a^2 \sigma \epsilon T^4 \]

absorbing & radiating areas

the same
(e) More general case

$$F_0 (1 - A) \pi a^2 = \pi a^2 \varepsilon T^4$$

with \(1 \leq \chi \leq 4\)

\[\uparrow \quad \uparrow\]

flat plate, isothermal sphere, planet = coolest case

= hottest case

\(T_{\text{isothermal sphere}} : T_{\text{isothermal hemisphere}} : T_{\text{flat plate}}\)

\[
\begin{align*}
(1) & : (2^{1/4}) : (2^{1/2}) \\
& = (1) : (1.2) : (1.4)
\end{align*}
\]

\(\chi\) is a complicated function of the rotation period, rotation pole direction, and thermal parameters, especially \(k \, [m^2 \cdot s^{-1}]\).

For most asteroids, comets it is unknown.

\(\chi = 1 \rightarrow \text{"subsolal" temperature.}\)
Even More General Case

Heating Rate = Cooling Rate

\[ f_0 (1 - \lambda) \pi a^2 = \chi \pi a^2 \sigma \varepsilon T^4 + \int f \left( \frac{2T}{\varepsilon r} \right) \frac{T}{\text{conduction term}} \]

+ this is already a complicated problem for a sphere, much more so for any other shape.

Really need to solve

\[ \int f_0 (1 - \lambda) \cos \theta \ dA = \int \sigma \varepsilon T^4 (\theta) \ dA + \int \text{conduction} \]

where \( \Theta = \text{sub-solar latitude} \)

\( \theta = 0 = \text{sun overhead} \)

- A local expert is Aurelie Guibert.
- Team Parge is also expert on temperature issues.
$\chi=2$ case is often called "STM" - the Standard Thermal Model.

STM is good to $\pm 5$ or $10\%$ in diameter, as judged by occultations.
Definitions

\[ Q_s = \frac{\text{actual scattering cross-section}}{\text{geometric cross-section}} = \frac{C_s}{\pi a^2} \]

\[ Q_a = \frac{\text{actual absorbing cross-section}}{\text{geometric cross-section}} = \frac{C_a}{\pi a^2} \]

Extinction \( Q_e = Q_s + Q_a \)

Single Particle Albedo

\[ \alpha = \frac{Q_s}{Q_s + Q_a} = \frac{Q_s}{Q_e} \]

Dimensionless size = "size parameter"

\[ \chi = \frac{2\pi a}{\lambda} \]

Optics depend a lot on \( \chi \)
Scattering

Consider single particle scattering:

1. Grazes the particle
2. Goes through the middle

Particle refr. index $n$

Wavelength in space $\lambda$

\[ \lambda = \frac{n}{\sqrt{n}} \]

Number of waves across particle diameter

\[ N_1 = \frac{2a}{\lambda} \]

\[ N_2 = \frac{2a \cdot n}{\lambda} \]

Difference

\[ \Delta N = \frac{2a (n-1)}{\lambda} \]

$n$ (Hi-C) $\sim 4/3$

$n$ (glass) $\sim 3/2$

$n$ (diamond) $\sim 5/2$
When $\Delta N$ = integer, the waves constructively interfere, i.e. when

$$\frac{2a(n-1)}{\lambda} = m \quad (m = 1, 2, 3, \ldots)$$

Set $m = 1$; 1st maximum when

$$x_1 = \frac{2\pi a}{\lambda} = \frac{m\pi}{(n-1)} = \frac{\pi}{n-1}$$

$m = 2$; 2nd maximum $x_2 = \frac{2\pi}{n-1}$

So, scattering efficiency looks like

$$Q_s$$

$Q_s \propto \frac{x}{x_2}$

$Q_s \propto \frac{1}{x}$

$Q_s \propto \frac{x}{x_2}$

eg: glass, $n \approx 3^{1/2}$, $x \approx \frac{\pi}{3^{1/2}} \approx 2\pi \approx 6$ or and

$\left\{
\begin{array}{l}
\text{for } x \ll 1 \ (a \ll 1) \text{ particle is invisible to } \\
\text{the wave, like a person to a tsunami}
\end{array}
\right.$
So, very small particles \( \ll 2\pi \) in the regolith do not individually matter much **BUT** they can be very abundant. (c.f. nanoFe)

The angular distribution of scattering by a particle also depends on \( x \).

\[ a < \lambda, \text{ nearly isotropic} \]

\[ a > \lambda, \text{ forward scatter} \]

**forward scatter angle**

\[ \theta \approx \frac{\lambda}{a} \text{ radians} \]

eg: \( \lambda = 0.5 \mu m \) (white light), \( a \approx 50 \mu m \)

\[ \theta \approx 0.5 \times 10^{-2} \text{ radian} \]

\[ \frac{50}{50} \approx \frac{1}{2} \]

(= more than dust)

**Single particle scattering depends on**

\( a/\lambda \), shape \& orientation, wavelength \& composition (= refractive index)

Mie Theory applies *only* to homogeneous spheres.
Multiple Scattering

Is horrendously complicated, and poorly understood.

Macroscopic parameters are invoked to describe planetary objects. Important ones are albedo, phase function.

Albedo

There are several albedos, often confused

Bond Albedo

\[ A = \frac{\text{scattered power}}{\text{incident power}} \left( \frac{[W]}{[W]} \right) \]

\(0 \leq A \leq 1\). \(A = 0\) - blackbody. \(A = 1\), perfect reflector

Monochromatic Bond

\[ A_\lambda = \frac{\text{scattered power at } \lambda}{\text{incident power at } \lambda} \]

\( A = \frac{\int_{0}^{\infty} A_\lambda d\lambda}{\int_{0}^{\infty} I_\lambda d\lambda} \)
For many purposes, e.g., energy calculations, \( A \) is what we need. But \( A \) is hard to measure, because photons can scatter in any direction, but we can usually only observe from one direction, or a few.

**Geometric Albedo**, \( P_a \)

\[
P_a = \frac{\text{Scattered flux @ 0° phase angle}}{\text{(Scattered flux @ 0° from Lambertian scatterer of same size & location)}}
\]

\( I_0 \)

Lambert

\[
I(\theta) \propto \cos \theta
\]
For a Lambert sphere,

\[
\text{the area of an annulus goes like}
\]

\[
dA = 2\pi r (rd\alpha) \cdot r \sin \alpha
\]

\[
dA = 2\pi r^2 \sin \alpha \, d\alpha \quad \text{(i.e. bigger at the limb)}
\]

So the dimming due to the \( \cos(\theta) = \cos(90 - \alpha) = \sin \alpha \)

matches the area increase due to projection towards the limb \( \propto \sin \alpha \)

ie: Lambert sphere = uniformly bright

(eg: ping pong ball)
It is hard to observe from $\alpha = 0^\circ$. So, a phase correction is needed.

\[ \frac{\Phi(\alpha)}{\Phi(0)} \]

\[ \theta \]

"Shape"

Geometric component

- $\alpha = 0^\circ$
- $\alpha = 90^\circ$

Macro

Scattering component

$\alpha = 0$ peak is due to shadow hiding + other effects e.g. - dew drop brightening

Micro

sphere

$\frac{1}{2}$ sphere

glass spheroid

shadow
A $\propto p$ are related

$$A = pq$$

$$q = 2 \int_0^{\pi} \frac{I(\alpha)}{I(0)} \sin\alpha \, d\alpha$$

but, $q$ is rarely known, and $p$ is usually known, if at all, at only one or a few wavelengths. $P_\lambda \rightarrow P_\nu$ ($\nu = 0.55 \mu m$)

$P_r$ ($\lambda = 0.65 \mu m$) etc

$q$ typically in the range 0.5 to 1.5
<table>
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<th>Geometric V (0.55μm) albedo</th>
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<tbody>
<tr>
<td>M</td>
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</tr>
<tr>
<td>V</td>
<td>0.65 (atmosphere)</td>
</tr>
<tr>
<td>E</td>
<td>0.38 (atmosphere)</td>
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<tr>
<td>Ma</td>
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<tr>
<td>Hydra</td>
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Dust Cross-section and Mass

Sphere

\[ C = \pi a^2 \]

\[ M = \frac{4}{3} \pi \rho a^3 \]

\[ M = \frac{4}{3} (\pi a^2) \rho a \]

or \( M \sim \rho a \cdot \pi \)

[\text{kg/m}^3 \times \text{m} \times \pi \text{m}^2]

where \( a \) = sphere radius

Can be generalised for a size distribution

\[ M \sim \rho \bar{a} C \]

where

\[ M = \int_{\frac{4}{3} \pi \rho a^3} n(a) da \]

\[ C = \int \pi a^2 n(a) da \]

\[ \bar{a} = \frac{\int a \cdot \pi a^2 n(a) da}{\int \pi a^2 n(a) da} \]

average radius weighted by scattering cross-section

\[ \bar{a} \downarrow 10^6 \quad \text{so} \quad C \uparrow 10^6 \]

\[ \bar{a} = 1 \text{m}, \quad C \approx 1m^2 \]

\[ \bar{a} = 1 \mu \text{m}, \quad C \approx 10^6 \mu \text{m}^2 = 1\text{km}^2 \]

\[ \text{eg:} \quad \text{1m ball pulverized to 1\mu m grains...} \quad M = \text{constant} \]
Space Weathering
Freshly pulveri