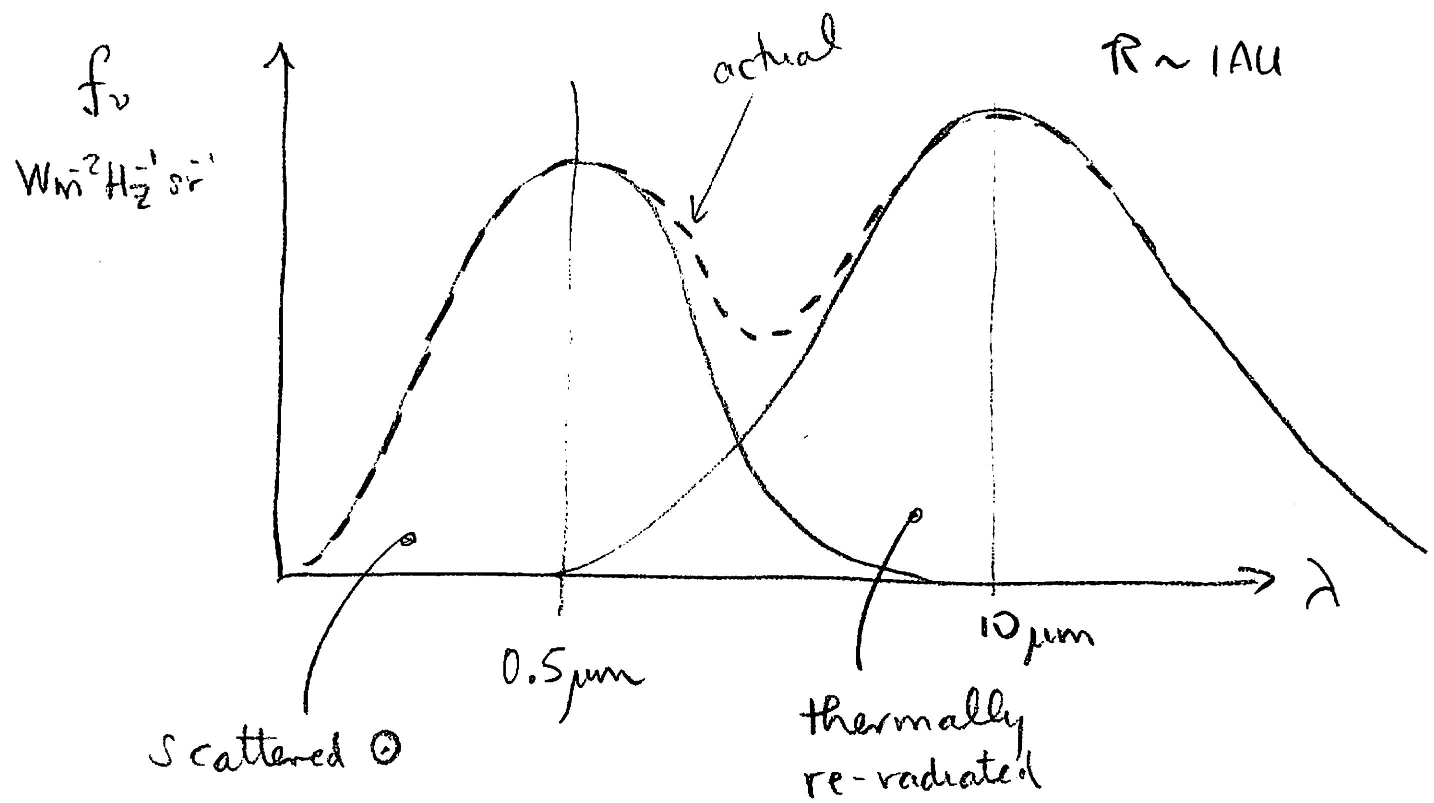
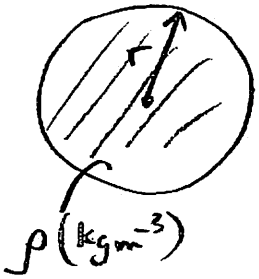
Planetary Surfaces

March 31 2011 D Jewitt

- Most surfaces are inaccessible & can be probed only with photons
- Most surfaces are porous aggregates
- ∴ we care about photons interacting with powders - a very tough subject.
- Photons can be scattered or absorbed. The absorbed ones heat the surface & lead to emission of longer λ photons



Temperature - Heat Transport



Heat energy $H = m \, c_p \, T$ [J]

sp. ht. capacity J kg^{-1} K^{-1}

mass

Temperature

Or $H \sim \rho r^3 c_p T$

Heat loss by conduction

conductivity W m^{-1} K^{-1}

$$\frac{dH}{dt} = r^2 \, k \, \frac{dT}{dr} \quad [\mathrm{J\,s^{-1}}]$$

center to edge temp gradient

Cooling Time $\tau \sim \dfrac{H}{\dot{H}} \quad \dfrac{[\mathrm{J}]}{[\mathrm{J\,s^{-1}}]}$

$$\sim \frac{\rho r^3 c_p T}{r^2 k \, (dT/dr)}$$

What to do about dT/dr?

Simplest approx is to set edge $T = 0$, $T = T_{middle}$

Then $\dfrac{dT}{dr} \sim \dfrac{T}{r}$

So $$\tau \sim \frac{\rho r^3 c_p \cancel{T}}{r^2 k \left(\cancel{T}/r\right)}$$

$$\sim \left(\frac{\rho c_p}{k}\right) \cdot r^2$$

$$\tau \sim \frac{r^2}{K}$$

where $K = \dfrac{k}{\rho c_p}$ = thermal diffusivity

$$[K] = \frac{\cancel{J} s^{-1} m^{-1} \cancel{K^{-1}}}{\cancel{kg} m^{-3} \cancel{J} \cancel{kg^{-1}} \cancel{K^{-1}}} = m^2 s^{-1} \text{ (area/time)}$$

eg: Pea: $r \sim 3\,mm$, $K_{pea} \sim \dfrac{1}{\underset{kg\,m^{-3}}{10^3}\ \underset{J\,kg^{-1}K^{-1}}{10^3}} \sim \underline{\underline{10^{-6}}}\ m^2 s^{-1}$

So $\tau_{pea} \sim \dfrac{(3 \times 10^{-3})^2}{10^{-6}} \sim \underline{\underline{10\ sec}}$

Potato; $r \sim 3\,cm$

$\tau_{potato} \sim \underline{1000\ sec}$

in agreement w/. experience

Again $$\boxed{\tau \sim \frac{r^2}{K}}$$

Distance travelled by heat through conduction in time τ is $r \sim \sqrt{K\tau}$

The thermal diffusivity of dielectric solids of planetary interest is $K \sim 10^{-6}\ m^2 s^{-1}$.
Metals are $\sim 100 \times$ larger. Powders are smaller, by factor strongly dependent on porosity. (eg $K \sim 10^{-7}$ or less).

eg: $\tau = 1$ day, $K = 10^{-6}\ m^2 s^{-1}$ (rocky planet)

$$\boxed{r \sim \sqrt{10^{-6} \times 10^{5}} \sim \sqrt{10^{-1}} = 0.3\,m}$$

Sun drives a T wave with scale 0.3m due to $\oplus$ rotation

[eg: hot sand under a cold night-time beach

Heat conduction equation (1D)

$$\frac{dT}{dt} = \overset{K}{\left(\frac{k}{\rho c_p}\right)} \cdot \frac{d^2T}{dr^2}$$

can be solved for a planet surface illuminated by sun + rotating. Behavior depends on $\underline{\underline{K}}$, not k, ρ, c_p separately.

Planetary scientists often use "thermal inertia",

$$\underline{I = \sqrt{k \rho c_p}} \qquad \left[J s^{-1} m^{-1} K^{-1} \, \cancel{kg} \, m^{-3} J \, \cancel{kg}^{-1} K^{-1}\right]^{1/2}$$

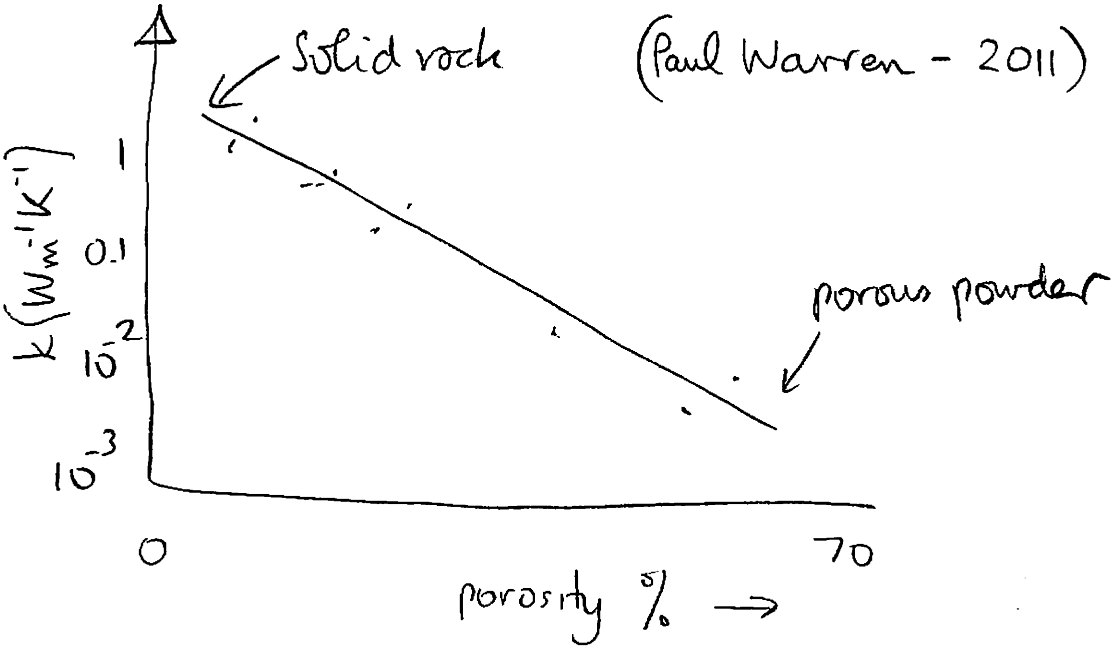
$$= kg^2 m^2 s^{-4} s^{-1} m^{-4} K^{-2}$$

$$* \left(I = \sqrt{K \rho^2 c_p^2} = \sqrt{K} \rho c_p\right) \qquad = kg \, m^{-1} s^{-5/2} K^{-1} = \text{very ugly}$$

eg: $I \sim \sqrt{1 \cdot 10^3 \cdot 10^3} \sim 10^3$ MKS units

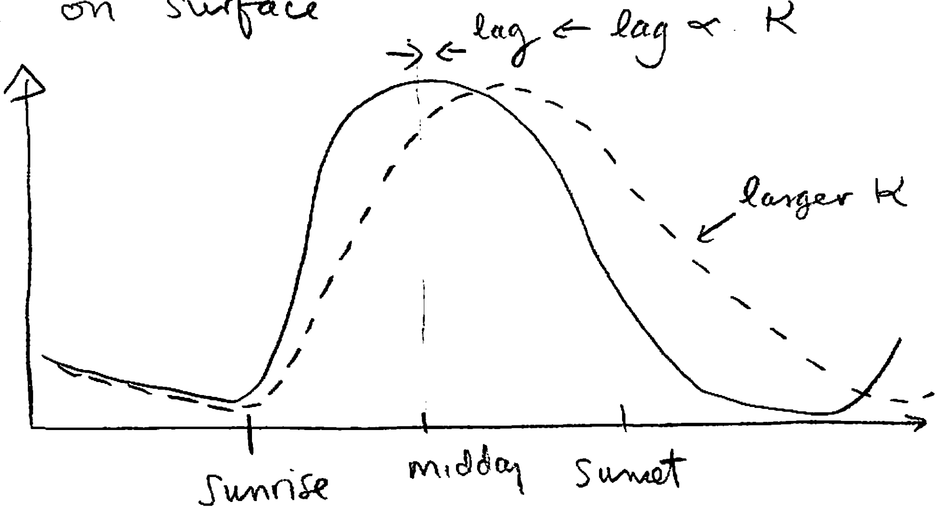
Fluffy regolith may have $I < 100$ MKS

Thermal inertia is not a happy thing - physically diffusivity is what matters.

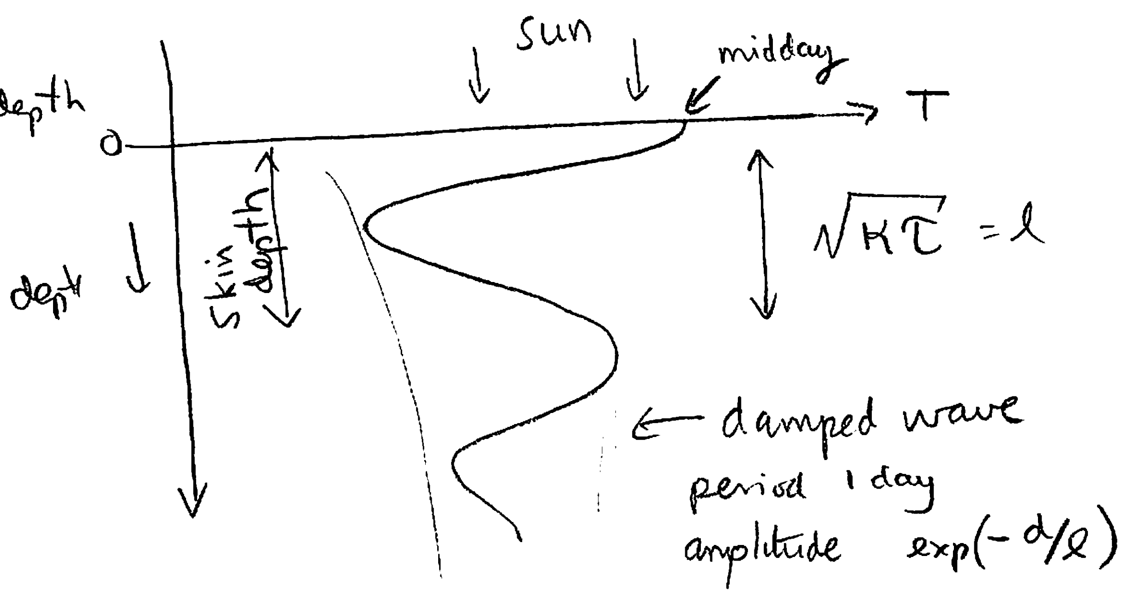


T vs time on surface

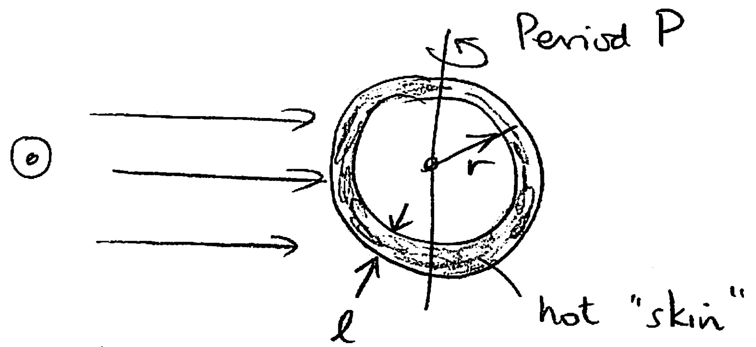
peak T $\neq$ noon



T vs depth



Diurnal Effects



Skin depth $\quad \ell \sim \sqrt{KP}$

Skin heat energy $\quad E = \underbrace{(4\pi r^2 \rho \ell)}_{\text{mass of shell}} c_p T$

Heat is radiated to space @ rate

$$\dot{E} = 4\pi r^2 \varepsilon \sigma T^4 \quad \text{(Black body)}$$

ε emissivity $(0 < \varepsilon < 1)$

$[\sigma = 5.67 \times 10^{-8}\ W m^{-2} K^{-1}]$

If rotation is fast, body will not have time to cool at night.

ie: ~ isothermal if $\quad E \gg \dot{E} P$

$$\cancel{4\pi} \cancel{r^2} \rho \sqrt{KP}\, c_p \cancel{T} \gg \cancel{4\pi} \cancel{r^2} \varepsilon \sigma T^{\cancel{4}\,3} P$$

Square $\quad \rho^2 K \cancel{P} c_p^2 \gg \varepsilon^2 \sigma^2 T^6 P^{\cancel{2}}$

or $\quad P \ll \dfrac{\rho^2 K c_p^2}{\varepsilon^2 \sigma^2 T^6}$

or $$P \ll \frac{\rho^{\cancel{2}}\, k\, \cancel{c_p}}{\varepsilon^2 \sigma^2\, \cancel{\rho}\, \cancel{c_R}\, T^6}$$

$$\boxed{P \ll \frac{\rho k c_p}{\varepsilon^2 \sigma^2 T^6}}$$

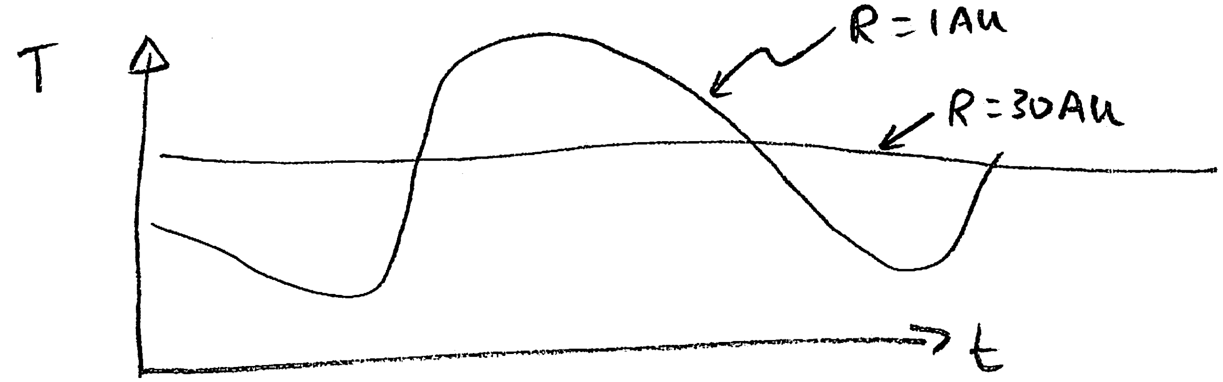
Substitute $$P \ll \frac{10^3 \cdot \underbrace{0.1}_{\text{regolith}} \cdot 1000}{1^2 \cdot (5.67\times10^{-8})^2\ (T/100)^6}$$

$$\boxed{P \ll \frac{8600\ \text{hr}}{(T/100)^6}}$$

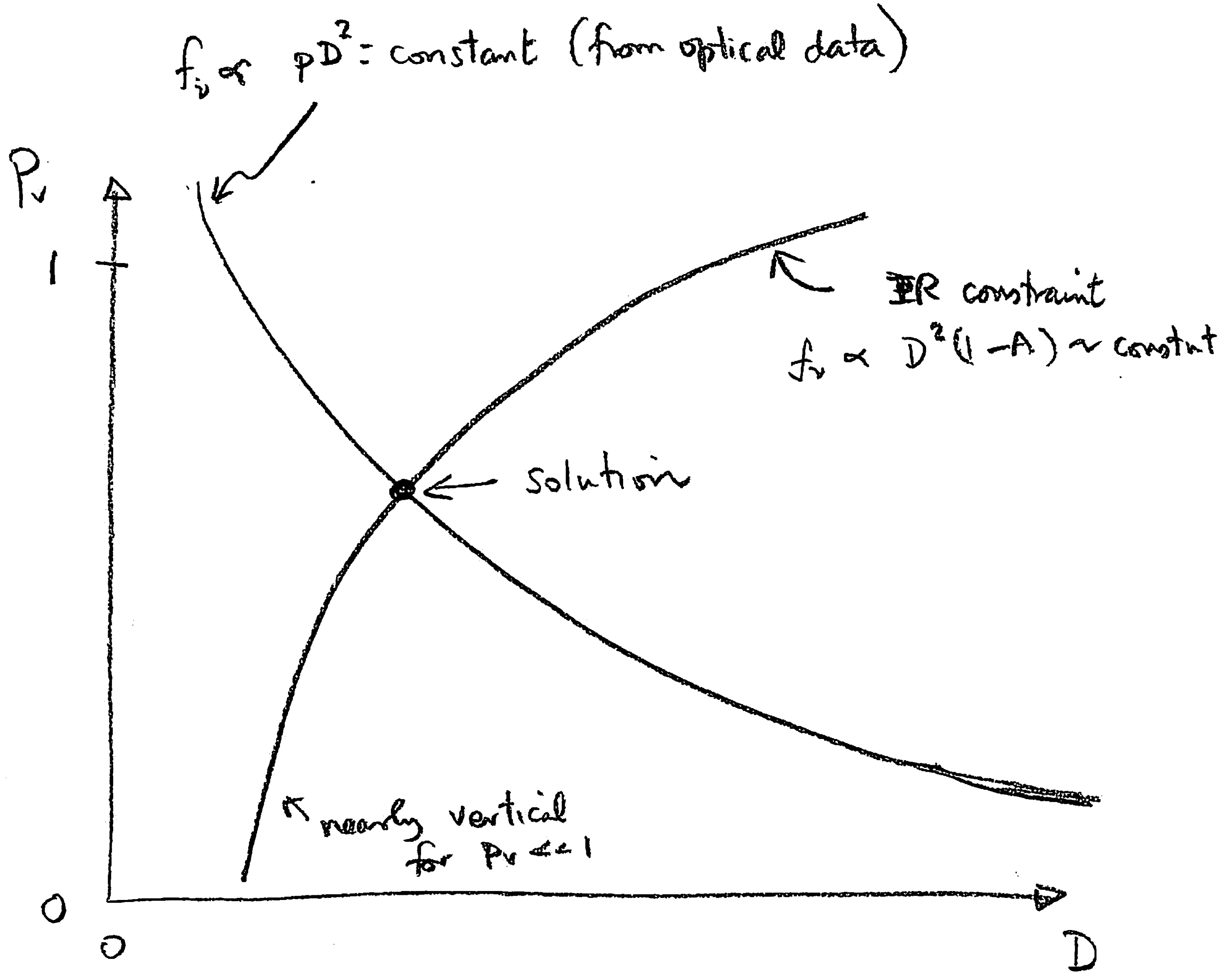
eg: $T = 100\,K$ ($R \sim 10$ AU - saturn) $P \ll 8600^{\text{hr}}$

$T = 300\,K$ ($R \sim 1$ AU, - earth) $P \ll 10^{\text{hr}}$

So, asteroids experience significant diurnal T variation (so does Moon), but KBOs do not.



for exactly the same object properties.

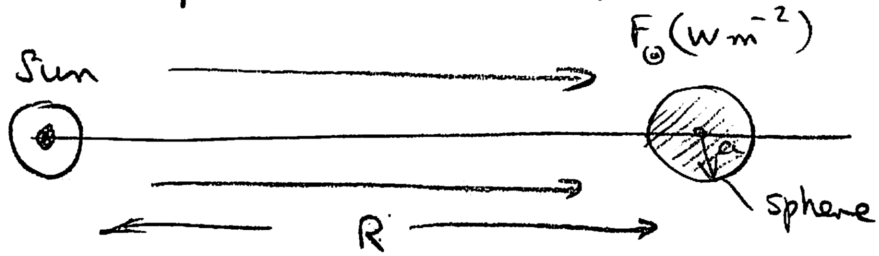


Jewitt + Kalas '97

[Graphical representation of the thermal method for determining asteroid size + albedo.]

Temperature

a) Black Body in Radiative Equilibrium



Heating Rate = Cooling Rate

$$F_\odot \cancel{\pi} \cancel{a^2} = 4\cancel{\pi} \cancel{a^2} \sigma T^4$$

substit → $$F_\odot = \frac{L_\odot}{4\pi R^2} \left[\frac{W}{m^2}\right]$$

$$T_{BB} = \left(\frac{L_\odot}{16\pi R^2 \sigma}\right)^{1/4} = \left(\frac{4 \times 10^{26}}{16\pi \cdot 5.67 \times 10^{-8}}\right)^{1/4} \frac{1}{\sqrt{R}}$$

express Δ in AU, $1\,AU = 1.5 \times 10^{11}\,m$, then

$$\boxed{T_{BB} = \frac{278}{R_{AU}^{1/2}} \; [K]}$$

eg: $R_{AU} = 1$, $T_{BB} = 278\,K$; $R_{AU} = 10\,AU$, $T_{BB} \sim 88\,K$

$$T_{BB} \propto \frac{1}{\sqrt{R}}$$

(b) Spherical Planet w/. Finite Albedo & Emissivity, but isothermal

$$F_\odot (1-A)\,\pi a^2 = \overset{\text{sphere - isothermal}}{4\pi a^2}\, \varepsilon\, \sigma\, T^4$$

A: Bond albedo

ε: emissivity $\quad 0 \le \varepsilon \le 1$

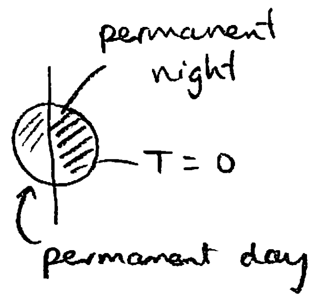
A = fraction of incident power that is absorbed.

$$\rightarrow \quad \boxed{T = \left(\frac{1-A}{\varepsilon}\right)^{1/4} T_{BB}}$$

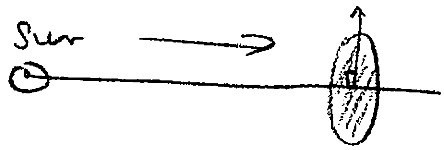
(c) Spherical planet, non-rotating, A, ε, no atmosphere

$$F_\odot (1-A)\,\pi a^2 = 2\pi a^2 \sigma \varepsilon T^4$$

radiating area 2x absorbing



(d) Flat plate planet



$$F_\odot (1-A)\,\underbrace{\pi a^2 = \pi a^2}\, \sigma \varepsilon T^4$$

absorbing & radiating areas the same

(e) More general case

$$F_{\odot}(1-A)\pi a^2 = \chi \pi a^2 \sigma \varepsilon T^4$$

with $1 \leq \chi \leq 4$

$\uparrow$ flat plate planet = hottest case

$\uparrow$ isothermal sphere = coolest case

$T_{\text{isothermal sphere}} : T_{\text{isothermal hemisphere}} : T_{\text{flat plate}}$

$$(1) : (2^{1/4}) : (2^{1/2})$$

$$= (1) : (1.2) : (1.4)$$

χ is a complicated function of the rotation period, rotation pole direction, and thermal parameters, especially $K\ [m^2 s^{-1}]$.

For most asteroids, comets it is unknown.

$\chi = 1 \rightarrow$ "subsolar" temperature.

(f) Even More General Case

Heating Rate = Cooling Rate

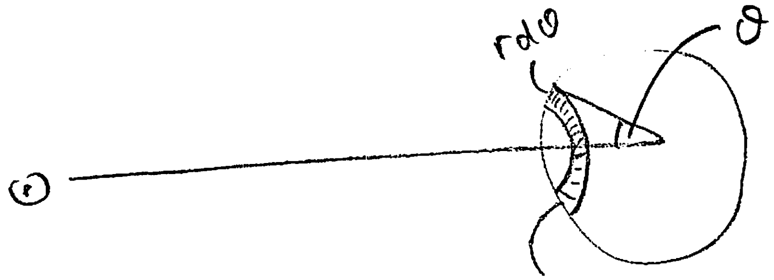
$$f_{\odot}(1-A)\,\pi a^2 = \chi\,\pi a^2 \sigma \varepsilon T^4 + f\left(\frac{\partial T}{\partial r}\right)$$

↑ conduction term

& this is already a complicated problem for a sphere, much more so for any other shape.

(g) Really need to solve

$$\int f_{\odot}(1-A)\cos\theta \, dA = \int \sigma \varepsilon T^4(\theta)\, dA + \int \text{conduction}$$

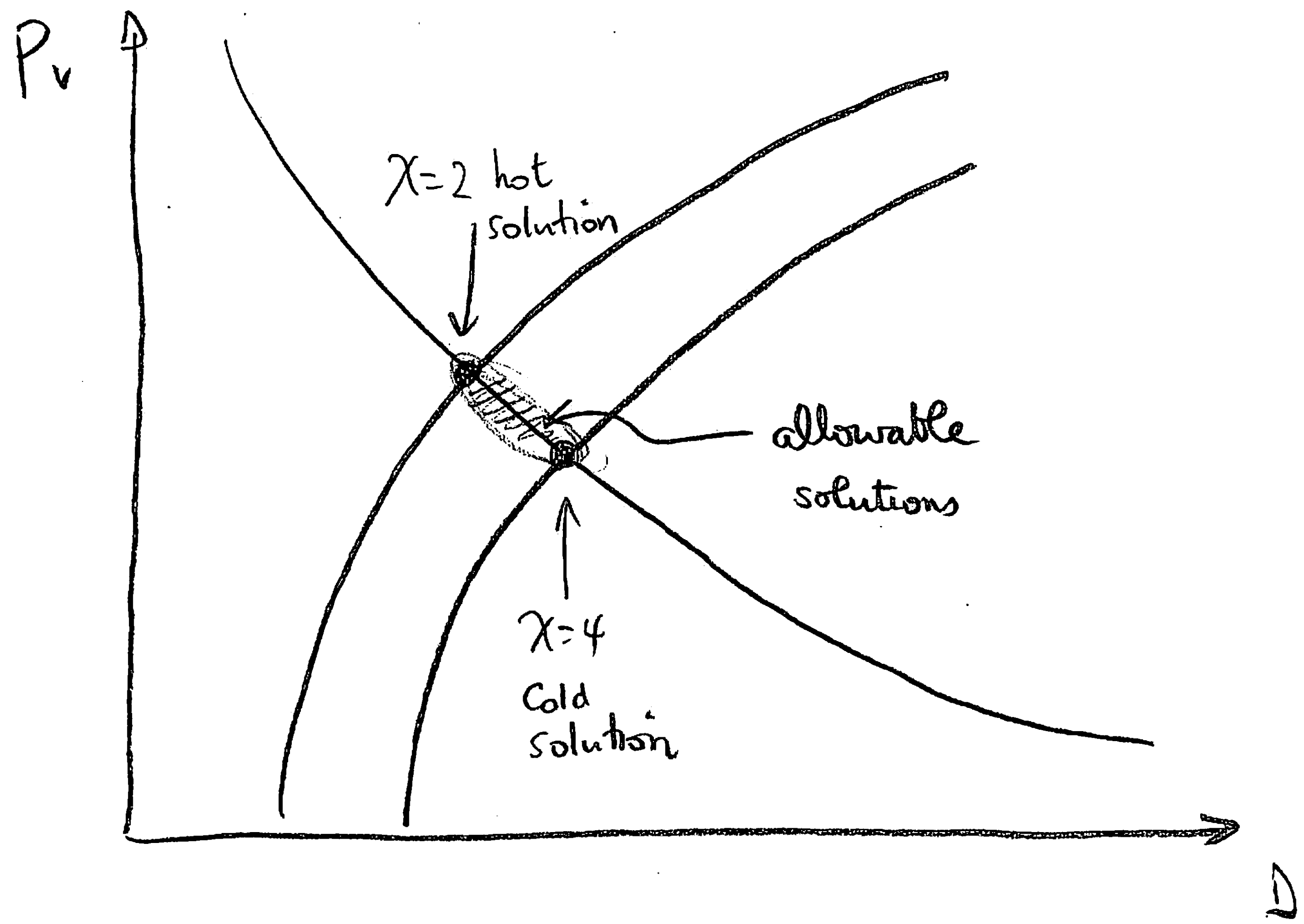


$dA = r\sin\theta \,.\, 2\pi \,.\, r\,d\theta$

where θ = sub-solar latitude

$\theta = 0$ = sun overhead

- A local expert is Aurelie Guilbert.
- Team Paige is also expert on temperature issues.

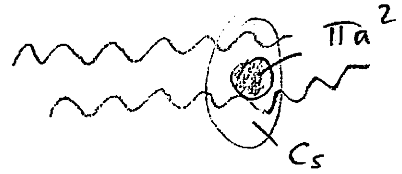


$\chi = 2$ case is often called "STM" –

the Standard Thermal Model.

STM is good to $\pm \sim 5$ or 10% in diameter, as judged by occultations.

Definitions



$$Q_s = \frac{\text{actual scattering cross-section}}{\text{geometric cross section}} = \frac{C_s}{\pi a^2}$$

$$Q_a = \frac{\text{actual absorbing cross-section}}{\text{geometric cross-section}} = \frac{C_a}{\pi a^2}$$

Extinction $Q_e = Q_s + Q_a$

Single Particle Albedo

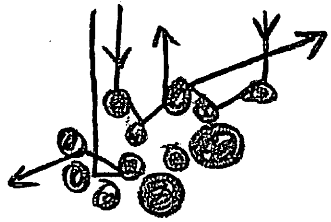
$$\alpha = \frac{Q_s}{Q_s + Q_a} \equiv \frac{Q_s}{Q_e}$$

Dimensionless size = "size parameter"

$$x \equiv \frac{2\pi a}{\lambda}$$

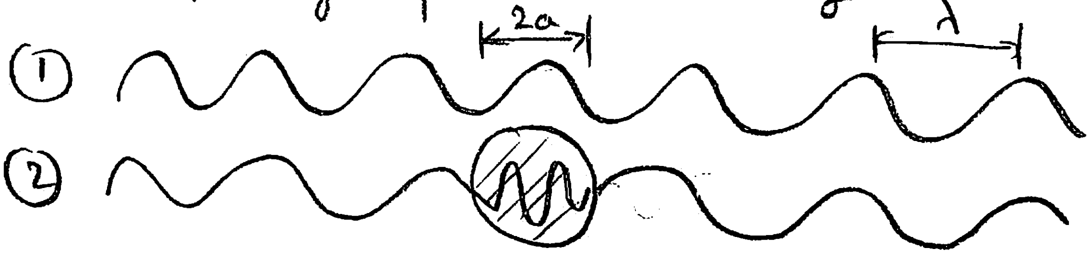
<u>Optics depend a lot on x</u>

Scattering



multiple scatt matters

Consider single particle scattering



(1) grazes the particle

(2) goes through the middle

$n(H_2O) \sim 4/3$
$n(\text{glass}) \sim 3/2$
$n(\text{diamond}) \sim 5/2$

Particle refr. index n.

Wavelength in space λ

" " particle $\dfrac{\lambda}{n}$

\# waves across particle diameter

$$N_1 = \frac{2a}{\lambda}$$

$$N_2 = \frac{2a \cdot n}{\lambda}$$

Difference $\Delta N = \dfrac{2a}{\lambda}(n-1)$

When ΔN = integer, the waves constructively interfere, ie when

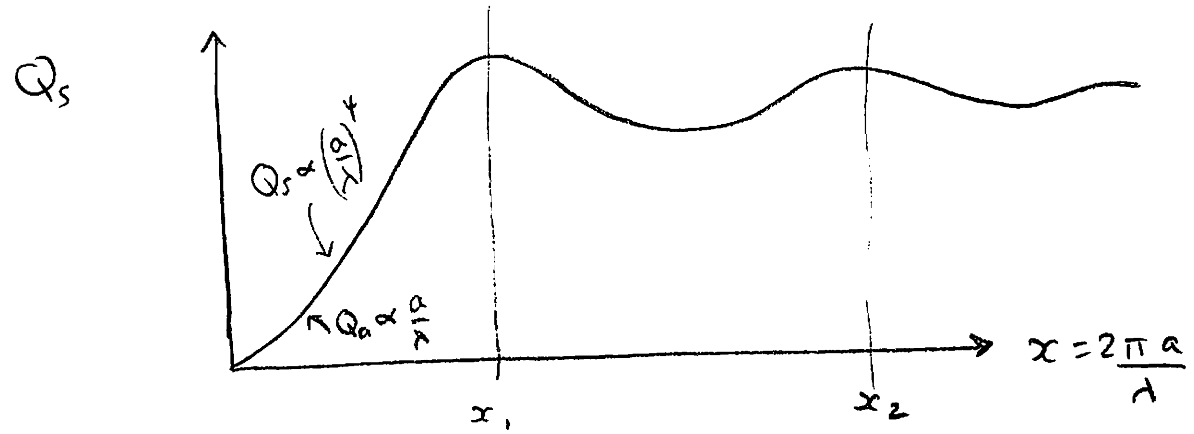
$$\frac{2a(n-1)}{\lambda} = m \qquad (m = 1, 2, 3 \ldots)$$

Set $m=1$; 1st maximum when

$$x_1 \equiv \frac{2\pi a}{\lambda} = \frac{m\pi}{(n-1)} = \frac{\pi}{n-1}$$

$m=2$; 2nd maximim $\quad x_2 = \frac{2\pi}{n-1}$

So, scattering efficiency looks like

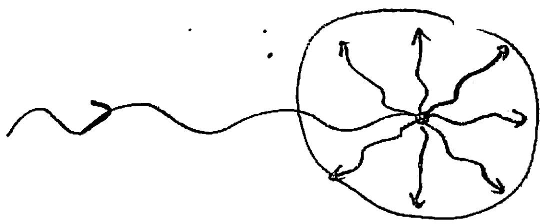


[eg: glass, $n \sim 3/2$, $x_1 \sim \frac{\pi}{\frac{3}{2}-1} \sim 2\pi \sim 6$ or $a \sim \lambda$

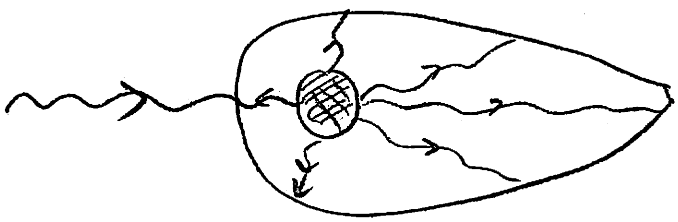
{for $x \ll 1$ $(a \ll \lambda)$ particle is invisible to the wave, like a person to a tsunami}

So, very small particles $x \ll 2\pi$ in the regolith do not individually matter much BUT they can be very abundant. (c.f. nano Fe)

The angular distribution of scattering by a particle also depends on x.



$a < \lambda$, nearly isotropic



$a > \lambda$ forward scatter

forward scatt angle

$\theta \sim \frac{\lambda}{a}$ radians

eg: $\lambda = 0.5\mu m$ (white light), $a \sim 50\mu m$

$\theta \sim \frac{0.5}{50} \sim 10^{-2}$ radian $\sim \frac{1}{2}^\circ$

(= movie ~~theater dust~~)

Single particle scattering depends on a/λ, shape + orientation, wavelength + composition (= refractive index)

Mie Theory applies ~ only to homogenous spheres.

Multiple Scattering

Is horrendously complicated, & poorly understood.

Macroscopic parameters are invoked to describe planetary objects. Important ones are albedo, phase function.

Albedo

There are several albedos, often confused

Bond Albedo

$$A = \frac{\text{scattered power}}{\text{incident power}} \left(\frac{[W]}{[W]} = [0] \right)$$

$0 \leq A \leq 1$. $A = 0$ – blackbody. $A = 1$, perfect reflector

Monochromatic Bond

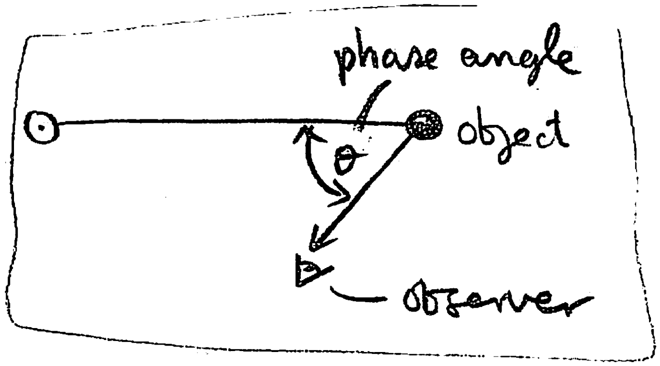
$$A_\lambda = \frac{\text{scattered power at } \lambda}{\text{incident power at } \lambda}$$

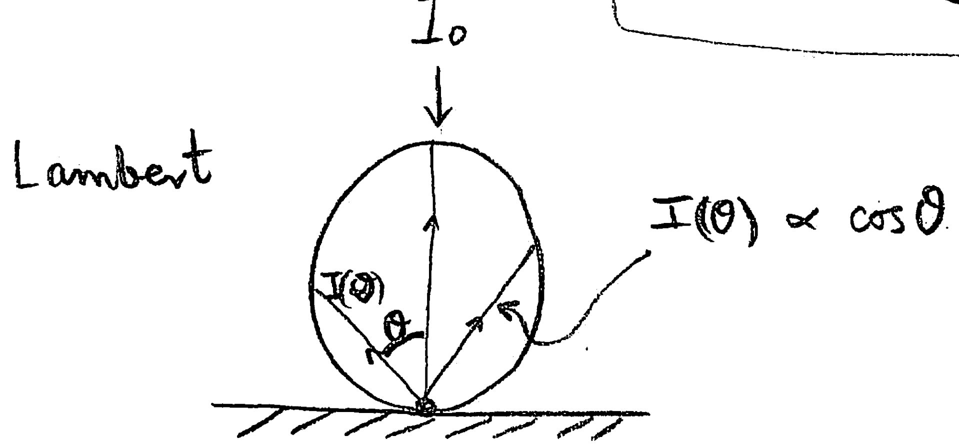
$$\left(A = \frac{\int_0^\infty A_\lambda \, d\lambda}{\int_0^\infty I_\lambda \, d\lambda} \right)$$

For many purposes, eg. energy calculations, A is what we need. But A is hard to measure, because photons can scatter in any direction but we can usually only observe from one direction, or a few.

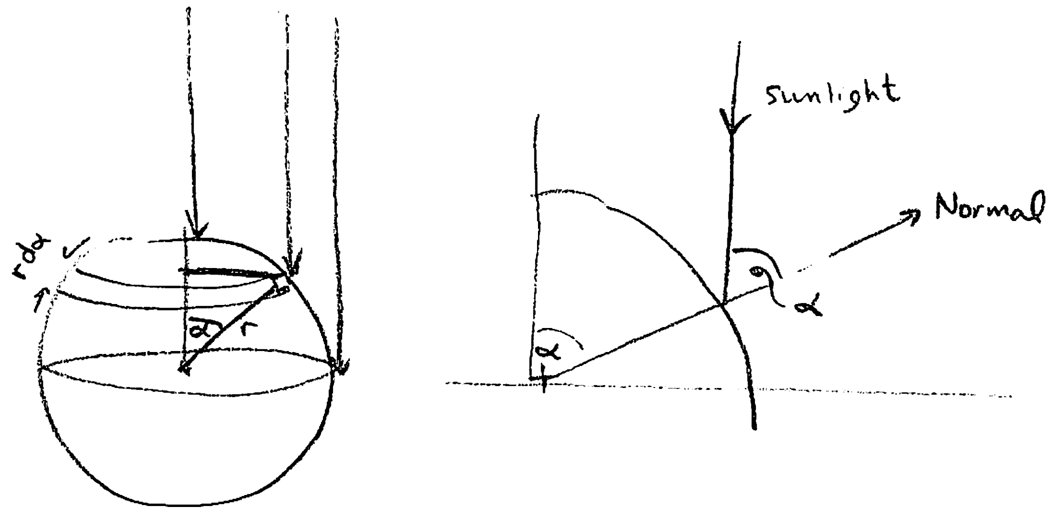
Geometric Albedo, p_λ

$$p_\lambda = \frac{\text{scattered flux @ } 0^\circ \text{ phase angle}}{(\text{scattered flux @ } 0^\circ \text{ from Lambertian scatterer of same size + location})}$$





For a Lambert sphere,



the area of an annulus goes like

$$dA = 2\pi (r\,d\alpha)\, r \sin\alpha$$

$$dA = 2\pi r^2 \sin\alpha \, d\alpha$$ (ie bigger at the limb)

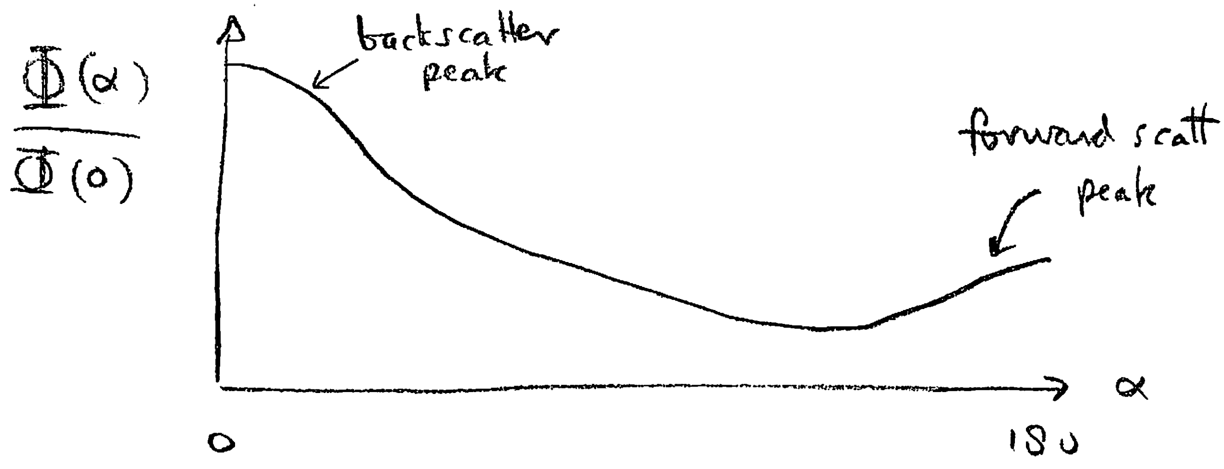
So the dimming due to the $\cos(\theta) = \cos(90-\alpha)$
$= \underline{\sin\alpha}$

matches the area increase due to projection towards the limb $\propto \underline{\sin\alpha}$

ie: Lambert sphere = uniformly bright

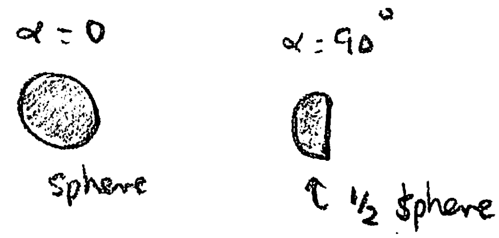
(eg: ping pong ball)

It is hard to observe from $\alpha = 0°$. So, a phase correction is needed



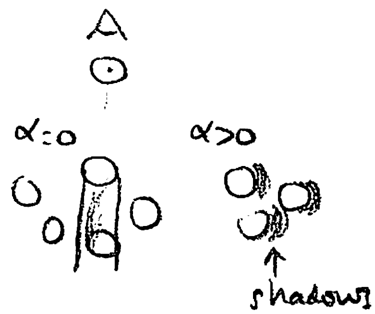
Geometric "shape" component

Macro

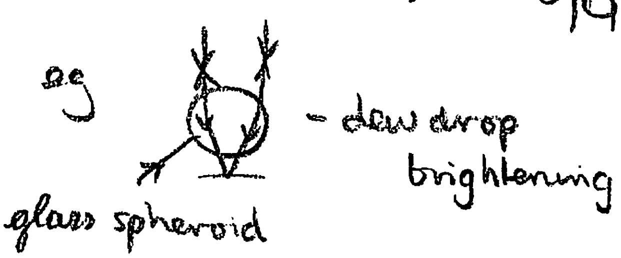


Scattering component

α = 0 peak is due to shadow hiding



+ other effects eg



micro

A & p are related

$$A = p\,q$$

$$q = 2\int_0^{\pi} \frac{I(\alpha)}{I(0)} \sin\alpha \, d\alpha$$

<u>but</u>, q is rarely known, and p is usually known, if at all, at only one or a few wavelengths. $P_\lambda \rightarrow P_V$ ($V = 0.55\,\mu m$)

P_R ($R = 0.65\,\mu m$) etc

(*) q typically in the range 0.5 to 1.5

Geometric V (0.55μm) albedo

M	0.11
V	0.65 (atmosphere)
E	0.38 (atmosphere)
Ma	0.15

Moon	0.12

Phobos	0.07
Deimos	0.07

Io	0.63
Europa	0.67
Ganymede	0.43
Callisto	0.17
Amalthea	0.09
irregulars	0.04

Mimas	0.6
Enceladus	1.0
Tethys	0.8
Dione	0.6
Rhea	0.6
Titan	0.2
Hyperion	0.3
Iapetus	0.6
Phoebe	0.08
Janus	0.6
Epimetheus	0.5

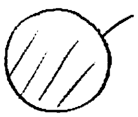
Ariel	0.39
Umbriel	0.21
Titania	0.27
Oberon	0.23
Miranda	0.32

Triton	0.72
Nereid	0.16
Naiad	0.07
Thalassa	0.09

Pluto	0.65
Charon	0.37
Nix	0.08
Hydra	0.18

ASIDE

Dust Cross-Section & Mass

 sphere

$$\left.\begin{aligned} C &= \pi a^2 \\ M &= \tfrac{4}{3}\pi \rho a^3 \end{aligned}\right\} \quad M = \tfrac{4}{3}(\pi a^2)\rho a$$

or $M \sim \rho a C \qquad [\text{kg}\,\cancel{m^{3}}\ \cancel{m}.\ \cancel{m^{2}}]$

where a = sphere radius

Can be generalised for a size distribution

$$\boxed{M \sim \rho \bar{a} C}$$

where $M = \int \tfrac{4}{3}\pi \rho a^3\, n(a)\, da$

$$C = \int \pi a^2\, n(a)\, da$$

and $\bar{a} = \dfrac{\int a.\ \pi a^2\, n(a)\, da}{\int \pi a^2\, n(a)\, da}$ = average radius weighted by scattering cross-section

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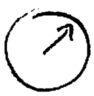
eg:  1m ball pulverized to 1 μm grains ... M = constant

$$\bar{a} \downarrow 10^6 \quad \text{so} \quad C \uparrow 10^6. \quad \begin{cases} a = 1\text{m}, & C \sim 1\,\text{m}^2 \\ a = 1\mu\text{m}, & C \sim 10^6\,\text{m}^2 = 1\,\text{km}^2 \end{cases}$$

Space Weathering

Freshly pulveri