Cometary Nuclei – Ultimate case of surface evolution

Basic model proposed by Whipple (1950) = "Dirty Snowball" model

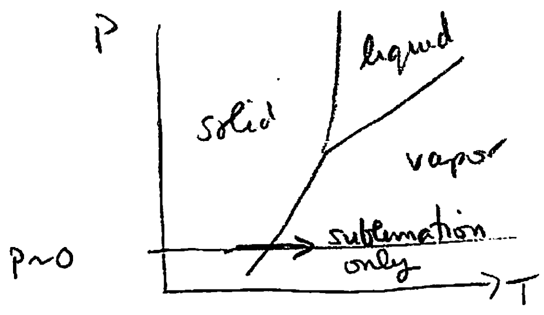
Heat balance equation

Define $f_\odot$ = solar constant = 1360 W m^{-2} @ 1 AU

Then flux @ another R is $\frac{f_\odot}{R_{AU}^2}$, where R_{AU} = heliocentric distance in AU.

$$\underbrace{\overset{W m^{-2}}{\frac{f_\odot}{R_{AU}^2}} (1-A) \overset{m^2}{\pi a^2}}_{\text{incident}} = \underbrace{\chi \overset{m^2}{\pi a^2} \overset{J s^{-1} m^{-2}}{\varepsilon \sigma T^4}}_{\text{radiated}} + \underbrace{\chi \overset{m^2}{\pi a^2} \overbrace{\overset{kg\, m^{-2} s^{-1}}{\dot{m}(T)} \overset{J\, kg^{-1}}{L(T)}}^{J s^{-1}}}_{\text{sublimation}}$$

where $\dot{m}$ (kg m^{-2} s^{-1}) = sublimation mass flux.
$L(T)$ (J kg^{-1}) = latent heat of sublimation



$$\dot{m} = \dot{m}(T)$$

Pressure = $\frac{d}{dt}$(momentum) ÷ area

$$P(T) = \dot{m}(T)\, V_{th}$$
$$= \dot{m}(T) \sqrt{\frac{2\pi k T}{\mu m_H}}$$

T = Temp, μ = molec wght (eg H_2O = 18), m_H = mass hydrogen atom

Values for P(T) and L(T) are determined experimentally for water & a few other ices. (eg: $L(T) \sim 2\times10^{6}\,J\,kg^{-1}$ for H_2O)

P(T) can also be obtained from Clausius-Clapeyron Equation

$$\frac{dP}{dT} = \frac{L}{T\,\Delta V}$$

ΔV = volume change upon transition between two phases, per unit mass

$\left(\frac{V_{solid}}{m} - \frac{V_{gas}}{m} \sim -\frac{V_{gas}}{m}\right)$

= gradient of the phase boundary on P-T plot.

For sublimation, $\Delta V \sim \frac{1}{\rho_{gas}}$ (since solid is v. dense ~ 0 volume)

so $$\frac{dP}{dT} = \frac{L\rho}{T}$$

and if the sublimated gas is a perfect gas

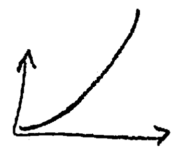
$$P = \frac{\rho kT}{\mu m_H} \quad \text{or} \quad \rho = \frac{P\mu m_H}{kT}$$

so $$\frac{dP}{dT} = \frac{LP\mu m_H}{kT^2}$$

$$\int\frac{dP}{P} = \int\left(\frac{\mu m_H L}{k}\right)\frac{dT}{T^2}$$

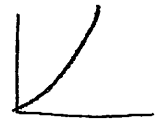
assume $L \neq L(T)$ $$\ln P = -\frac{\mu m_H L}{kT}$$

$$P = P_0 \exp\left(\frac{-\mu m_H L}{kT}\right) \qquad T\downarrow,\ P\uparrow\uparrow$$



so $$\dot{m} = P_0 \exp\left(\frac{-\mu m_H L}{kT}\right)\sqrt{\frac{\mu m_H}{2\pi kT}}$$

$T \to 0$, $\dot{m} \to \exp(-\infty)$ $\qquad \propto \dfrac{\exp\left(\frac{-1}{T}\right)}{\sqrt{T}}$



τ v. rapid $f(T)$ - faster than exponential

Anyway, you get the point.

Rough solutions & notes

① T is reduced by sublimation - a natural refrigerator

because energy is sucked-up by breaking bonds

② Near sun, σT^4 term $<<$ $\dot{m} L$ term (because sublimation v. strong @ high temperatures). so

$$\frac{F_\odot}{R_{AU}^2}(1-A)\cancel{\pi}\cancel{a^2} \sim \chi \cancel{\pi}\cancel{a^2}\,\dot{m} L$$

so $$\boxed{\dot{m} \sim \frac{F_\odot (1-A)}{\chi L R_{AU}^2} \propto \frac{1}{R_{AU}^2}}$$

eg: $f_\odot = 1360\ \mathrm{W\,m^{-2}} \sim 10^3\ \mathrm{W\,m^{-2}}$

Take $R_{AU} = 1$, $A \sim 0$, $L(H_2O) \sim 2\times10^6\ \mathrm{J\,kg^{-1}}$, $\chi \sim 1$

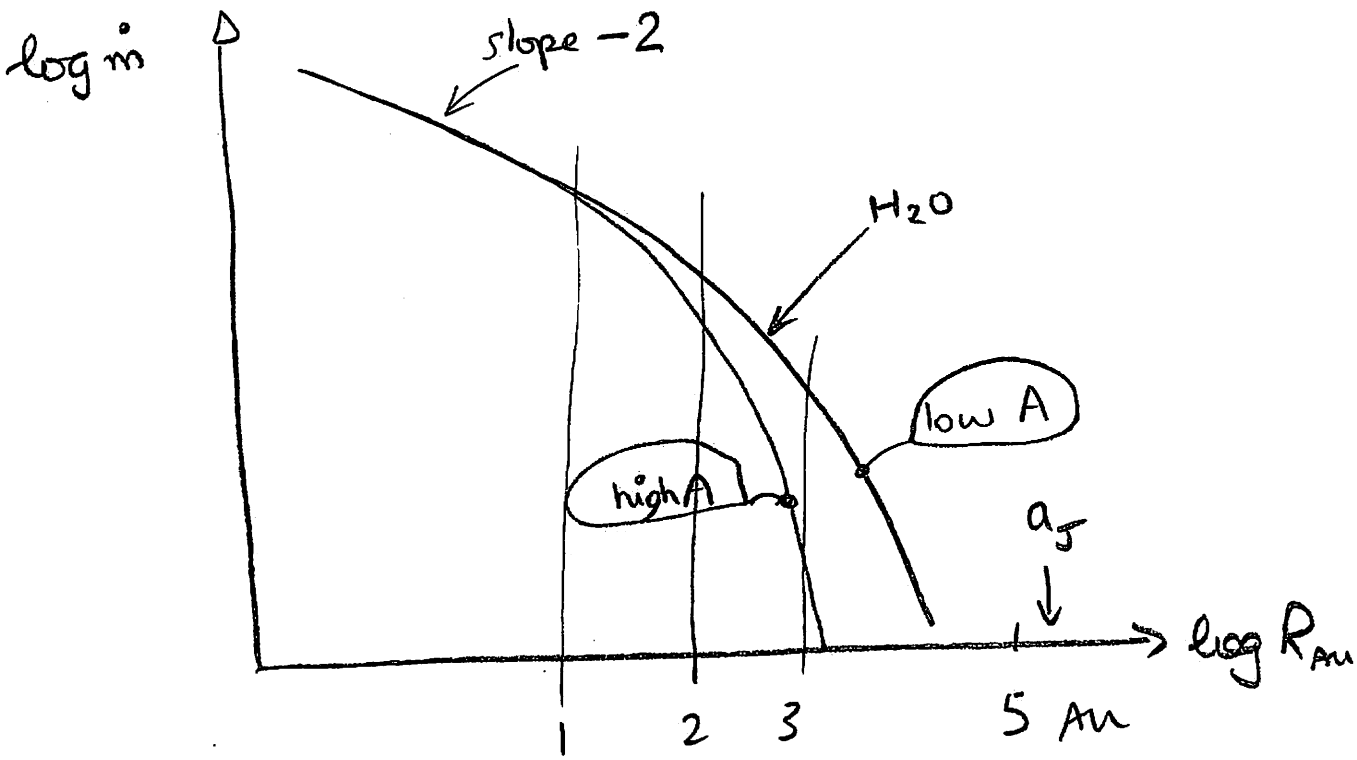
$$\dot{m} \sim \frac{1000}{2\times10^6} \sim 5\times10^{-4}\ \mathrm{kg\,m^{-2}\,s^{-1}}$$

$\therefore$ $1\ \mathrm{km}^2 = 10^6\ \mathrm{m}^2$ produces $5\times10^{-4}\times10^6 \simeq 500\ \mathrm{kg\,s^{-1}}$

$\sim$ 0.5 tonne/second

Typical comets produce few 100 to few 1000 $\mathrm{kg\,s^{-1}}$.

Extreme comet Hale Bopp produced $10^5\ \mathrm{kg\,s^{-1}}$.



(*) H_2O sublimation negligible for $R > R_J \sim 5.2$ AU

Space Shuttle "ice."

Nucleus Lifetime

Nucleus of size "a", sublimating from πa^2 will lose

1a $$\frac{dM}{dt} = \chi \pi a^2 \dot{m} \propto a^2 \dot{m}$$

and "sublimation lifetime" $\tau_s = \dfrac{M}{dM/dt} \sim \dfrac{\rho a^3}{a^2 \dot{m}} \sim \dfrac{\rho a}{\dot{m}}$

eg: $\rho = 1000 \text{ kg m}^{-3}$, $a = 500\text{ m}$, $\dot{m} = 5\times10^{-4} \text{ kg m}^{-2}\text{s}^{-1}$

$$\tau_s \sim \frac{5\times10^5}{5\times10^{-4}} \sim 10^9 \text{ s} \sim 30 \text{ yrs @ 1 AU}$$

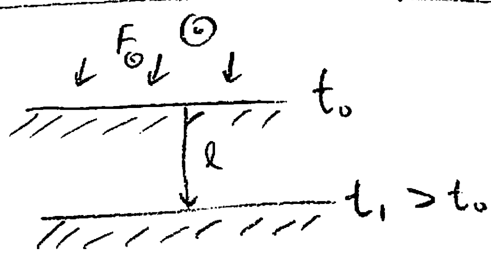
Non-circular + larger orbit → longer τ_s, but still

$\tau_s \ll 4.5$ Gyr.

∴ Comets must be recent arrivals in the inner solar system

→ Sources Kuiper Belt + Oort Cloud

Topo. evolution



$$\frac{dl}{dt} = \frac{\dot{m}}{\rho} \left[\frac{\text{kg m}^{-2}\text{s}^{-1}}{\text{kg m}^{-3}}\right]$$

$$\sim \frac{5\times10^{-4}}{10^3} \sim 0.5\ \mu\text{m/sec} = 15 \text{ m/yr}$$

(*) [Expect rapid evolution of surface topography by mass loss.

Mass Ejection

Sublimated H_2O has $V_g = \sqrt{\dfrac{kT}{\mu m_H}} \gg V_e = \sqrt{\dfrac{2GM_n}{r_n}}$

eg: $\begin{pmatrix} r_n = 1\,km, \\ \rho \equiv 1000\,kg\,m^{-3} \end{pmatrix}$ $V_e = \sqrt{2 \times 6.6 \times 10^{-11} \times \frac{4\pi}{3} \times 1000} \cdot 1000$

$\sim \sqrt{50 \times 10^{-8}} \cdot 10^{3} \sim 7 \times 10^{-4} \times 10^{3} \sim 0.7$

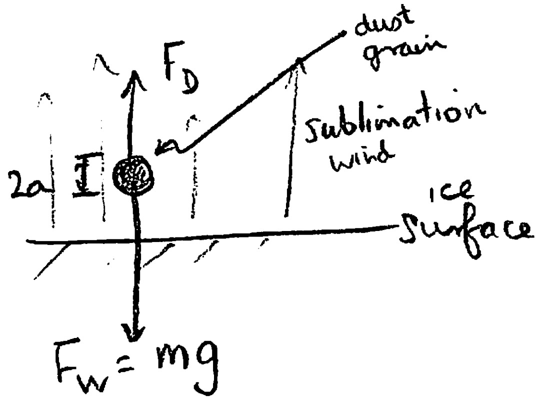
$\sim 1\ ms^{-1}$

and $V_g \sim \sqrt{\dfrac{1.38 \times 10^{-23} \times 200}{18 \times 1.67 \times 10^{-27}}} \sim \sqrt{10^{4} \cdot \dfrac{14}{1.7}} \sim 300\ ms^{-1}$

$V_g \gg V_e$

Gas immediately escapes.

Qu: What about solids?



Escape for $F_D \gg mg$

(# gas/volume)

eg: $F_D \sim C_D \pi a^2 . N_1 . \Delta V^2 . \mu m_H$

$m^2 \quad m^{-3} \quad m^2 s^{-2} \quad kg \quad = kg\,m\,s^{-2}$
$= \text{force}$

$\Delta V = V_{gas} - V_{dust}$

$N_1 = \dfrac{\dot{m}\ (kg\,m^{-2}s^{-1})}{\mu m_H \ V_g}$ (m^{-3})

$(kg) \quad (m\,s^{-1})$

Assume $\Delta v \sim v_{gas}$ (ie: slow dust) to make it easy.

Then $F_D \sim C_D \, \pi a^2 \, \frac{\dot{m}}{\cancel{\mu m_H} \cancel{v_g}} \cdot v_g^{\cancel{2}} \cancel{\mu m_H} = C_D \pi a^2 \dot{m} \, v_g$

C_D: drag coefficient ~ 1

and weight force is

$$F_W = \frac{4}{3}\pi \rho a^3 \, \frac{G M_n}{r_n^2}$$

$$= \frac{4}{3}\pi \rho a^3 \, G \frac{4}{3}\pi \rho_n r_n$$

$$= \frac{16\pi^2}{9} G \rho \rho_n r_n a^3$$

Escape occurs for $F_D \gg F_W$

$$C_D \cancel{\pi} \cancel{a^2} \dot{m} v_g \gg \frac{16\pi^{\cancel{2}}}{9} G \rho \rho_n r_n a_c^{\cancel{3}}$$

or $$a_c \ll \frac{9 C_D \dot{m} v_g}{16\pi G \rho \rho_n r_n}$$

$$\frac{\cancel{kg} m^{-2} \cancel{s^{-1}} \, m \cancel{s^{-1}}}{\cancel{kg} m^3 \cancel{s^{-2}} \cancel{kg^{-2}} m^{-3} \cancel{kg} m^{-6} m} = m^{-2+1-1-2+6-1} = [m]$$

phew!

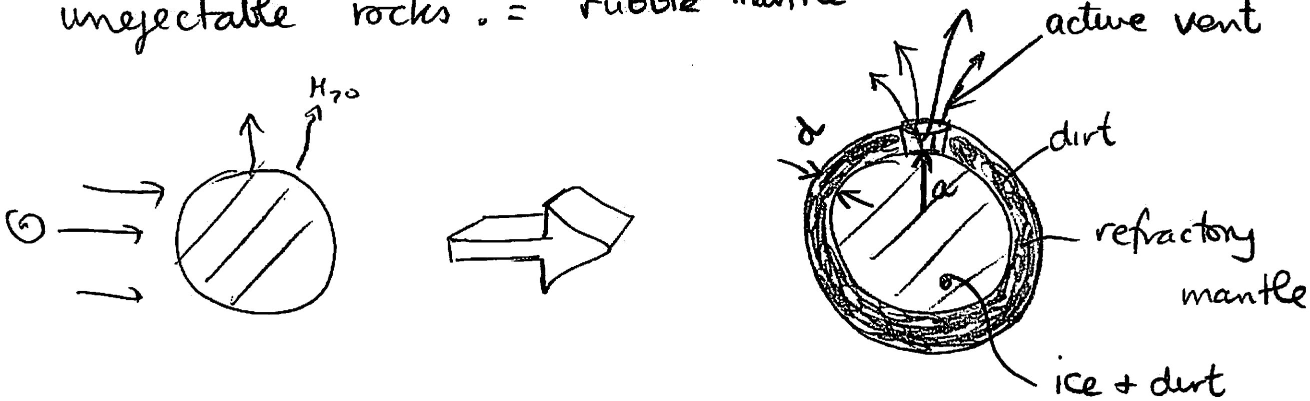
eg: $R_{AU} = 1$, $\dot{m} = 5\times10^{-4}\ kg\,m^{-2}s^{-1}$, $v_g \sim 300\ m s^{-1}$, $r_n = 1000\,m$

$$a_c \ll \frac{9 \times 1 \times 5\times10^{-4} \times 3\times10^{2}}{6.6\times10^{-11} \times 10^3 \times 10^3 \times 10^3 \times 16\pi} \sim \frac{20\times10^{-2}}{16\pi \; 10^{-2}}$$

$\simeq$ ½ meter

⊛ Potentially very large particles can be launched.

Material not lost falls back, creating a mantle of unejectable "rocks". = "rubble mantle"



Mantle stifles sublimation

a) by thermally insulating the ice

b) by inhibiting escape of sublimated molecules.

Strong effect when mantle thickness

$$d \gtrsim \text{skin depth} \sim \sqrt{\kappa P_{rot}}$$

th diffusivity

rotation period

eg. $\kappa = 10^{-7} m^2 s^{-1}$ (porous regolith)
$P_{rot} \sim 10$ hr
$\Big\}$ $\underline{d \sim 6 \text{ cm}}$

eg. $P = P_{orbit} = 10$ yr $\rightarrow$ $\underline{d \sim 5 m}$

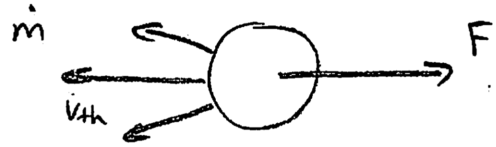
Both $\ll a$.

In practice, sublimation proceeds through active "vents" in a largely inactive mantle.

The vents throttle mass loss & determine τ_s..... perhaps extending it by factors of many.

Other Effects of Mass Loss

(a) **Rocket Effect:** Recoil due to asymmetric mass loss.



eg: mass loss $\frac{dM}{dt}$ ($kg\,s^{-1}$)

speed of ejection

$$\text{force} = \frac{d}{dt}(\text{momentum}) = \frac{dM}{dt} V_{th}$$

mass × accⁿ = Force

$$\frac{4}{3}\pi \rho a^3 \alpha = \frac{dM}{dt} V_{th} = \dot{m} \pi a^2 V_{th}$$

neglect 4, π, 3

$$\boxed{\alpha \sim \frac{\dot{m} V_{th}}{\rho a}} \quad m s^{-2}$$

if uni-directional = collimated

eg: @ 1AU

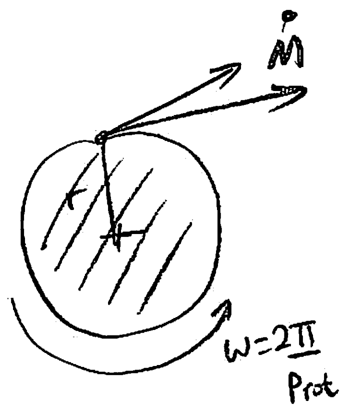
$$\alpha \sim \frac{5\times10^{-4} \times 300}{10^3 \times 500_m} \sim 3\times10^{-7}\ m s^{-2}$$

$$\left(\text{cf } g_\odot = \frac{GM_\odot}{1 AU^2} = 0.006\ m s^{-2}\right) \qquad \therefore \frac{\alpha}{g_\odot} \sim 10^{-4}$$

Very small, but over long times this "non-gravitational acc^n" is dynamically important.

[eg: $x = \frac{1}{2}\alpha t^2$, $\rightarrow$ $t = 1\,yr$, $x = \frac{1}{2} \times 3\times10^{-7} \times (3\times10^7)^2$
$= 10^8\,m = 100{,}000\,km$
$=$ large]

(b) <u>Torque Effect</u>: Non-central mass loss exerts a torque & can change spin.



$$\text{Torque} = \underset{\sim}{r} \times \underset{\sim}{F} \simeq k r F = \text{rate of change of ang. mom}, \underset{\sim}{L}$$

$$|\underset{\sim}{T}| = k r F = k r \left(\frac{dM}{dt} V_{th}\right)$$

$$\sim k\, r\, \dot{m}\, r^2\, V_{th}$$

↑ constant $0 < k < 1$ for non-tangential mass loss

k = dimensionless moment arm $0 < k < 1$

Ang Mom $\quad L = I\omega$, $\quad \omega = \frac{2\pi}{P_{rot}}$

[Moment of Inertia] $\quad I = \frac{2}{5} M r^2$ homogeneous sphere

$$T = \frac{dL}{dt} = \frac{2}{5} M r^2 \frac{d\omega}{dt}$$

$$\frac{2}{5} M r^2 \frac{d\omega}{dt} = k \dot{m} r^3 V_{th}$$

put $M \sim \rho r^3$ & neglect 2, 5, 4, π, 3

$$\rho r^5 \frac{d\omega}{dt} = k \dot{m} r^3 V_{th}$$

Spin up time $\tau_{spin} \sim \frac{\omega}{(d\omega/dt)} \sim \frac{2\pi}{P_{rot}} \frac{\rho r^2}{k \dot{m} V_{th}}$

$\propto r^2$, all-else equal.

= important for small nuclei

$$\left\{ \text{eg. } \tau_{spin} \sim \frac{2\pi \; 1000 \; . \; 1000^2}{(10 \times 3600) \times k \times 5 \times 10^{-4} \times 300} \sim \frac{10^6}{k} s \sim \frac{10 \text{ days}}{k} \right.$$

I calculate $k \sim 1/20$, so $\tau_{spin} \sim 200$ days @ 1 AU for 1 km nucleus $\}$

$\underline{\tau_{spin} << \tau_s \text{ and } << 4.5 \text{ Gyr}}$

τ_{spin} is probably the limiting timescale for small comets, leading to their rapid rotational breakup

{ c.f. Hartley 2, Tempel 1 science: H2 → $\frac{dP}{dt} \sim 1$ min/day & $P \sim 17^{hr} \sim 1000$ min }

Big Picture

Inside $R \sim 5$ AU, sublimation leads to

(1) mantle formation

(2) mass loss & topographical modification & orbital evolution

(3) shape evolution

(4) spin evolution

(5) destruction

Effects are interdependent, timescales maybe short

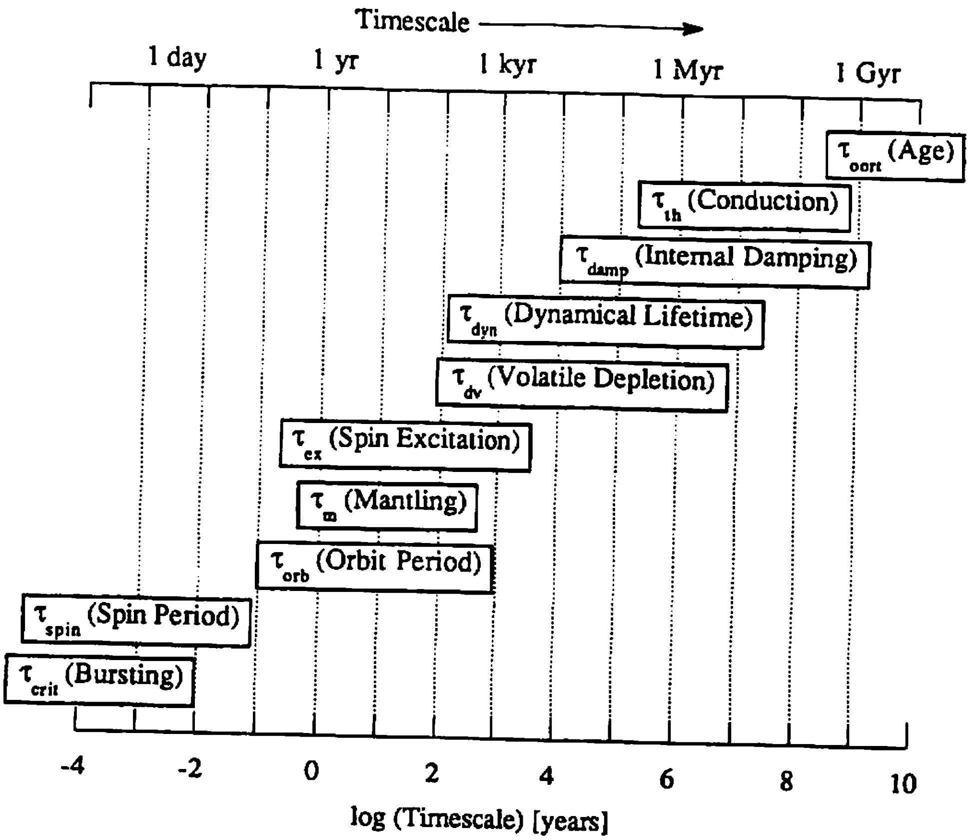


Figure 1 *Fundamental timescales appropriate to a 5 km radius short period comet. See discussion in §1.*

which are completely baked out of the near surface layers by solar heating. Conversely, $\tau_{th} < \tau_{oort}$ implies that the deep interiors retain no direct memory of the cometary formation temperatures.

- Second, $\tau_{dv} < \tau_{dyn}$ would seem to imply that nuclei can lose their entire volatile contents even before being ejected from the solar system by planetary perturbations, presumably leaving behind "asteroids". However, the second inequality $\tau_m << \tau_{dv}$ suggests that rapid growth of surface mantles can choke nuclear mass loss, and possibly preserve subsurface volatiles even in outwardly "dead" comets (§6).
- Third, $\tau_m < \tau_{dyn}$ implies that cometary mantles adjust to long term changes in the insolation. If true, the mass loss will always be regulated by a surface mantle (§5).
- Fourth, $\tau_{damp} < \tau_{oort}$ implies that excited rotational states resulting from formation will be absent in the present day comets. However, $\tau_{ex} < \tau_{dyn} < \tau_{damp}$ implies that precessional motions subsequently induced by torques due to mass-loss cannot be damped

Amorphous Ice

At low T, ice naturally forms amorphous (disordered).

Am ice spontaneously transforms to crystalline ice on timescale

$$\tau_{CR} = 3\times10^{-21} \exp\left(E_A/kT\right) \text{ (yrs)}$$

w/. $E_A/k = 5370\,K$

eg: $T = 260\,K$ (refridge) $\rightarrow$ $\tau_{CR} \sim 3\times10^{-12}$ yr $=$ 83 μs.

eg. $T = 130\,K$ $\longrightarrow$ $\tau_{CR} \sim 3\times10^{-3}$ yr $=$ 1 day

eg $T = 77\,K$ $\longrightarrow$ $\tau_{CR} = 4.6$ Gyr

$\therefore$ Ice w/. $T < 77\,K$ can remain Am for entire age of solar system.

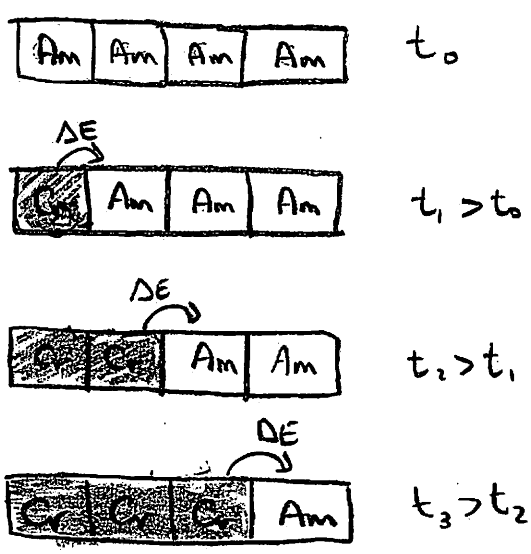
$\left\{ \text{By } T_{BB} \sim \frac{279}{R^{1/2}},\ T=77 \text{ corresponds to } R \sim 13\text{ AU} \right\}$

Ice in outer solar system could/should be Amorphous.

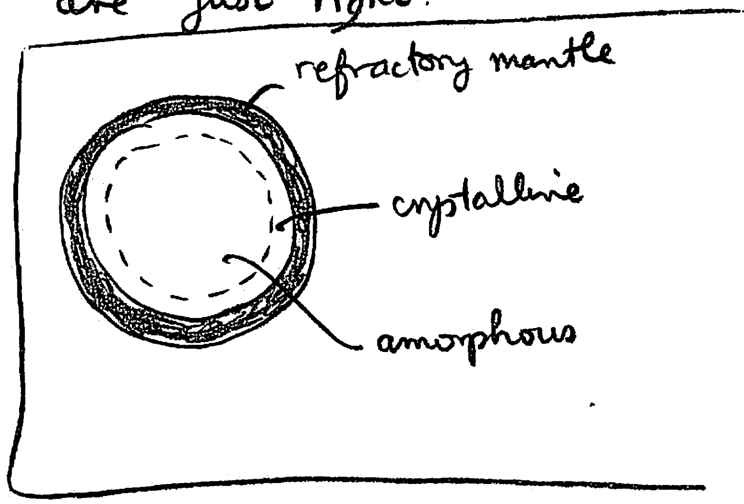
Transformation Am $\rightarrow$ Cr is exothermic $\underline{\Delta E \sim 10^5}$ J kg^{-1}

ΔE heats newly crystallized ice.

Thermal Runaway

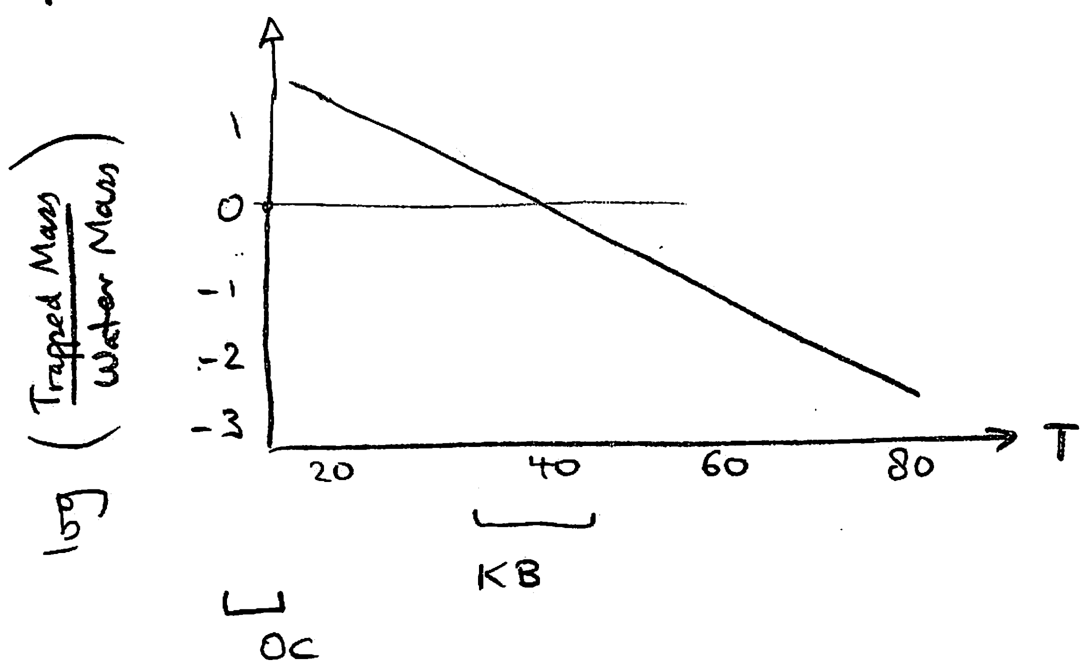


- Heating caused by Am → Cr can trigger more crystallization & a runaway, if conditions are just right.



Trapping

Am ice has v. large specific area, enabling it to trap other molecules.



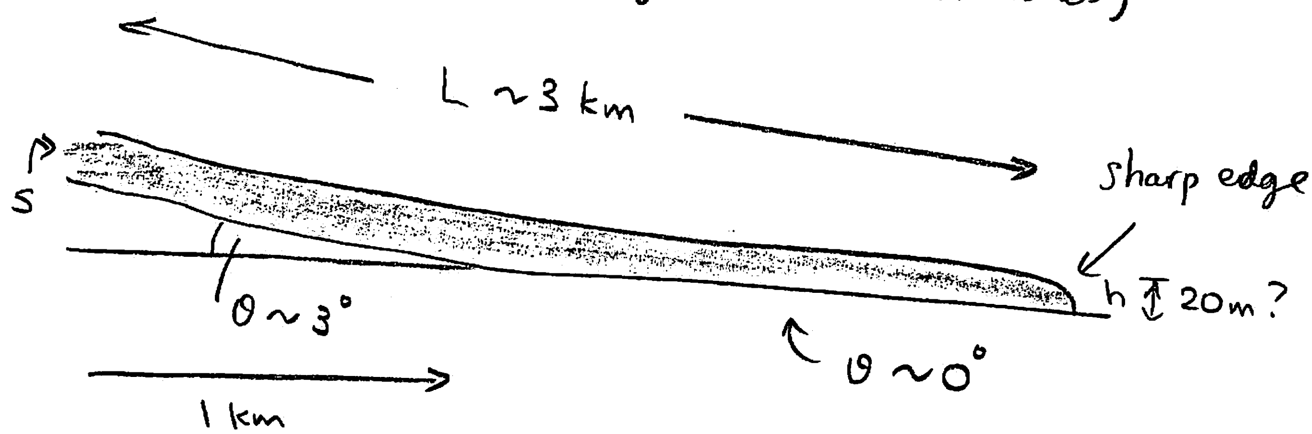
Am → Cr (a) releases ΔE

(b) frees the trapped gas

∴ Am ice invoked to explain many ice phenomena in comets & outer solar system bodies.

Tempel 1 Smooth Terrain models

(as an indication of current uncertainties)



$g \sin\theta \simeq 2\times10^{-4}\left(\frac{3}{60}\right) \simeq \underline{10^{-5}\,ms^{-2}}$ S = source (scale ~ 100m?)

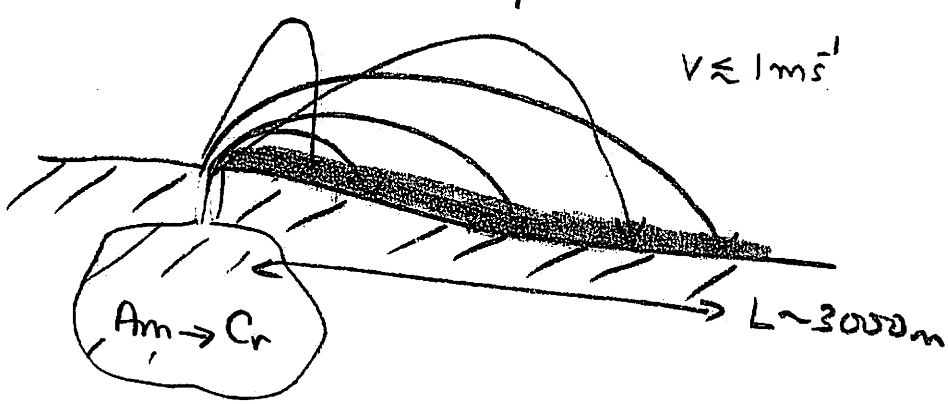
Width ~ 1 km. Area ~ 5 km^2, $\rho \sim 400\ kg\,m^{-3}$

mass $\boxed{m \sim \rho A h \sim 400 \times (5\times10^6)\times 20 = 4\times10^{10}\ kg}$

<u>Models</u> (1). Viscous flow Qu: of what? Under $g\sin\theta = 10^{-5}\,ms^{-2}$?

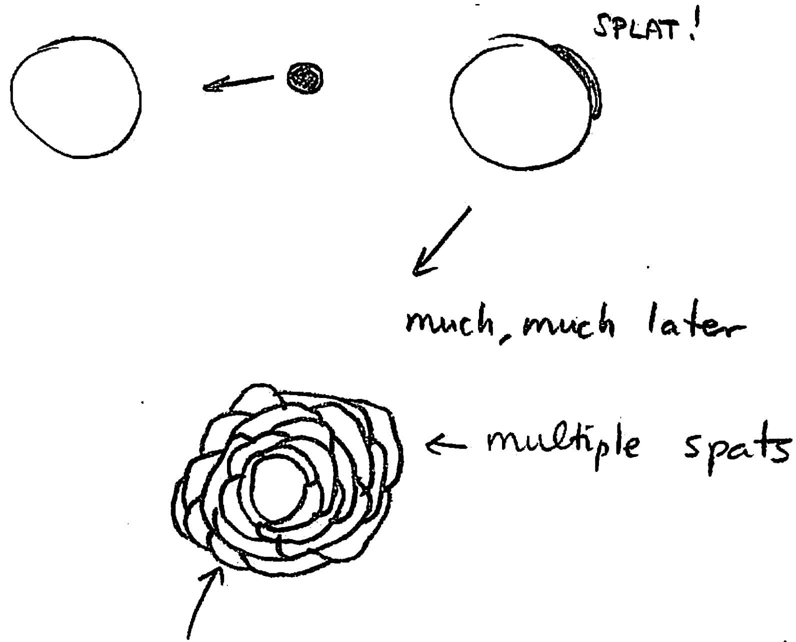
ν. limit set by requiring flow speed < $\underline{V_e \sim 1\,ms^{-1}}$.

(2). Ballistic deposit



Problem – ballistic deposit would have tapered end, not a sharp end

3 TALPS = SPLAT



smooth flow mass $m \sim 10^{10} - 10^{11}$ kg equivalent to a sphere of radius

$$r_e \sim \left(\frac{m}{\rho}\right)^{1/3} \sim 300 - 600\,\mathrm{m}$$

$$\#\ \text{Splats} \sim \frac{M_{nucleus}}{m} \sim 300 - 1000$$

Problems – smooth flows are localised on T1 & not seen at all on other nuclei like Wild 2. Should they not be general?

– not clear that a planetesimal would flatten like this.

– <u>need quantitative work</u>