Homework 1. Crater0size scaling with crustal strength of planetary bodies: Determine the role of pressure-dependent frictional strength in controlling the crater size and excavation volume on the surface of the Moon, Mars, Venus, and Earth.

Question 1. In the strength regime, the π scaling law of Holsapple (1993) has the following form:

$$V \propto \frac{m}{\rho}\left(\frac{\rho U^2}{Y}\right)^{\frac{3\mu}{2}}\left(\frac{\rho}{\delta}\right)^{1-3\nu}$$

where V is the volume of a crater formed by a given impact that has a vertical incident impact velocity U, a density δ, and a mass m. The target planetary body has a yield strength Y and a density ρ. Exponents $\nu \approx \mu \approx 1/3$.

Assume:

(1) The crustal strength of the Moon, Mars, and Earth follows the classic Coulomb fracture criterion, with the cohesive strength to be small and thus negligible.
(2) The angle of internal friction for the crust of the three bodies is the same, i.e., $\phi = 30°$ or the coefficient of internal friction $\mu_\phi = tan30°$.
(3) Only the strength of the upper 10-km crust of the planetary bodies affects the excavation volume of the craters.

Use the lithostatic pressure (i.e., p = density x surface gravity acceleration x depth) to approximate normal stress components on all potential brittle failure surfaces during the impact, please determine how the crater volume scales with the surface gravitational acceleration (i.e., even in the strength regime!) among the three planetary bodies using the above scaling relationship. The surface gravitational accelerations of the Moon, Mars and Earth are 1.63, 3.69, and 9.8 m/s^2, respectively; their crustal densities are 3.4 g/cm^3 (Moon), 3.4 g/cm^3 (Mars), and 3.0 g/cm^3 (Earth), respectively.

Question (2). For a wet planetary body whose crust is saturated with water, the effective coefficient of international friction can be expressed as

$$\overline{\mu}_\phi = \mu_\phi(1-\lambda)$$

where $\overline{\mu}_\phi$ is the effective coefficient of internal friction, μ_ϕ is the coefficient of internal friction, and λ is the Hubbert-Rubey pore-fluid-pressure ratio (usually taken as the ratio between the density of water and hosting rock). Please repeat the above question by comparing the crater scaling relationship for the Venus and Earth; the former is believed to be dry while the latter is definitely wet. The two planets are similar in size and have similar surface gravitational acceleration, can you tell which planet would have a larger crater volume hit by the same impactor?