COMETARY ROTATION: AN OVERVIEW

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Abstract. I discuss our current understanding of the spin states of cometary nuclei. Cometary spin influences the temporal and spatial patterns of outgassing from the nucleus (through diurnal and seasonal effects) and is in turn influenced by outgassing-driven torques. The current challenge to cometary astronomers is to quantify the interactions between the spin, the outgassing, and the resultant torques on the nucleus, and to understand the role of rotation in determining the basic physical properties of the nucleus.

Keywords: Comets, nucleus, rotation

1. Introduction

I aim to provide a background to the subject of cometary rotation in a style that is deliberately simplified in order to illuminate the key physical issues. The review is divided into 3 main parts. First, I develop the physical context for studies of cometary rotation in an attempt to convey why this subject is interesting and potentially important to cometary science. Second, I summarise existing evidence for the rotational characteristics of the nuclei of comets. Third, I examine particular constraints on the rotation of the nucleus of comet Hale–Bopp. To aid the interested reader, the review ends with a comprehensive list of references to (mostly) recent work on the rotational properties of comets. Previous reviews on the subject of nucleus rotation include those by Sekanina (1981a), Whipple (1982), and Belton (1991).

2. Basic Ideas: Important Timescales

It has long been recognized that the nuclei of comets spin, and that nuclear rotation may have observable effects on the photometric and morphological properties of the inner coma. It is also suspected that torques due to non-central outgassing may act to change the nucleus spin vector, leading to a change in the magnitude of spin period and excited rotational states. The interrelations between the spin and the properties of the nucleus are potentially very complicated. In order to clarify these processes, it is convenient to consider fundamental timescales relevant to the spinning cometary nucleus (Jewitt, 1992).



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The spin period of any body formed by accretion will be larger than the critical period

$$\tau_c(\text{hours}) = a \left(\frac{1000}{\rho}\right)^{1/2} \tag{1}$$

at which the centripetal acceleration at the surface of the nucleus equals gravitational acceleration towards the center (since accretion is strongly inhibited at periods $\leq \tau_c$). Here, ρ (kg m⁻³) is the nucleus density and *a* is a parameter that depends on the shape of the nucleus: a = 3.3 hours for a spherical nucleus, a = 6.6hours for a symmetric binary, and increases with elongation of the nucleus (Jewitt and Meech, 1988).

The energy of rotation of a body is $E = L^2/(2I)$ where $L = I\omega$ is the scalar angular momentum, I (kg m²) is the moment of inertia and $\omega = 2\pi/P$ (s⁻¹) is the angular spin rate corresponding to rotation period P (s). In the absence of external torques, L = constant, and the minimum energy of rotation is attained when the moment of inertia is maximized, corresponding to rotation about the short axis of the body (so-called "principal axis rotation"). In general, the newly formed nucleus will spin with its angular momentum vector and axis of maximum moment of inertia misaligned, and is said to be in an "excited state". Excited state rotation produces periodic stresses in the bulk of the nucleus, leading to frictional dissipation of energy and gradual re-alignment to principal axis rotation on the damping timescale (Burns and Safranov, 1973)

$$\tau_{\rm damp} \approx \frac{\mu Q}{\rho K_3^2 r_n^2 \omega^3}.$$
(2)

Here, μ (N m⁻²) is the rigidity, Q is the quality factor (fractional loss of energy per cycle), K_3 is a shape-dependent numerical factor and r_n (m) is the mean radius. The damping parameters appropriate to cometary nuclei are not well known. We follow Harris (1994) and take $\mu Q = 5 \times 10^{11}$ (N m⁻²) and $K_3^2 \approx 0.03$ (based on data for Phobos). Substituting $\rho = 10^3$ kg m⁻³, P = 10 h, we obtain $\tau_{damp} = 10^8$ (1 km/ r_n)² (yr) and accept that this estimate may be in error by an order of magnitude. A 5 km radius nucleus has $\tau_{damp} \sim 4 \times 10^6$ yr. This is much less than the age of the solar system (4.6 $\times 10^9$ yr) so that memory of the initial spin state should have been lost in a body this large. It is also less than the few $\times 10^7$ yr timescale for dynamical transfer from the Kuiper Belt to the inner solar system (Levison and Duncan, 1997) so that short-period comets (SPCs) produced recently by Kuiper Belt collisions (Farinella and Davis, 1996) should arrive in the inner solar system with fully damped rotational states (Giblin and Farinella, 1997).

Once in the inner solar system, cometary outgassing creates torques that change the angular momentum of the nucleus, either in the magnitude or the direction of the spin, or both (Figure 1a). Notice that the creation of a torque, T, does not require a morning/afternoon "thermal lag" type asymmetry in the outgassing rate;

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Figure 1. (a) Two vents on an irregular nucleus losing mass towards the sun (bottom of the figure). Recoil forces on the nucleus about the center of mass (black circle) exert a torque. The net torque on the nucleus is the sum of torques from all vents. (b) In response to the net torque, the spin of the nucleus evolves towards the critical frequency, ω_c (Equation (1)).

torque is naturally produced by any asymmetric distribution of vents on the nucleus. To within a numerical factor, the fractional change in the spin angular velocity resulting from mass loss is

$$\frac{\Delta\omega}{\omega} = k_T \left(\frac{\Delta M}{M}\right) \left(\frac{V_{\rm th}}{V_{\rm eq}}\right),\tag{3}$$

where $k_T \sim 0.05$ is the dimensionless moment arm (see Appendix A), V_{th} is the outgassing velocity, $V_{\text{eq}} = r_n \omega$ is the equatorial velocity and $\Delta M/M$ is the fractional change in the mass. Purely tangential mass loss from a spherical nucleus would have $k_T = 1$, while purely central mass loss would have $k_T = 0$. Equation (3) may be re-written to give the *e*-folding timescale for the change of angular momentum, $\tau_{\text{ex}} \approx L/T$ or

$$\tau_{\rm ex} \approx \frac{\omega \rho r_n^4}{V_{\rm th} k_T \frac{\mathrm{d}M}{\mathrm{d}t}},\tag{4}$$

where dM/dt is the net mass loss rate from all vents, V_{th} is the outflow speed and r_n is the effective nucleus radius (cf. Samarasinha et al., 1986; Jewitt, 1991).

All other things being equal, the total mass loss rate scales with the cross sectional area of the nucleus, as well as inversely with the square of the distance from the sun. Based on measurements of cometary outgassing, we take $dM/dt = 10^3 (r_n/1 \text{ km})^2$ (kg s⁻¹) at 1 AU, so that $\tau_{ex} \propto r_n^2$ by Equation (4). This relation is approximate, but reflects the mass loss rates measured for SPCs to at least one order of magnitude. With $k_T = 0.05$, P = 10 h, $\rho = 10^3$ kg m⁻³, $V_{th} = 10^3$ m s⁻¹, Equation (4) reduces to $\tau_{ex} = 0.1 (r_n/1 \text{ km})^2$ (yr). Notice that τ_{ex} is measured in

units of years of solar exposure at 1 AU. The typical earth-crossing short period comet might spend only 10% of each orbit inside or near 1 AU, so that the elapsed time for changing the spin is nearer $10\tau_{ex}$. The canonical 5 km radius nucleus would have $\tau_{ex} \sim 2.5$ yr at R = 1 AU. Strictly, Equation (4) holds only provided $\tau_{vent} > \tau_{ex}$, where τ_{vent} is the lifetime of the vent (or vents) driving the spin-up. When $\tau_{ex} > \tau_{vent}$ the spin-up will random walk towards ω_c as different vents turn on and off. However, the accumulation of random torques is very slow compared to the constant jet case, and it is unlikely that nuclei would reach ω_c within their dynamical lifetimes if $\tau_{ex} > \tau_{vent}$. Little direct information is available concerning the lifetimes of vents. However, jet patterns and non-gravitational accelerations in comets P/Halley, P/Swift–Tuttle and others appear to recur on successive orbits and suggest that vents survive for many (100s?) orbits (Sekanina, 1990; cf. Samarasinha and Belton, 1995).

Finally, we note that the timescale for de-volatilization of the cometary nucleus is (again, in units of solar exposure at 1 AU) of order

$$\tau_{\rm dv} = \frac{\rho r_n^3}{\frac{{\rm d}M}{{\rm d}t}} \tag{5}$$

which, with the above relation for dM/dt gives $\tau_{dv} = 100 (r_n/1 \text{ km})$ (yr). Equation (5) is really a lower limit to the timescale for volatile depletion, because the growth of a refractory mantle may choke the gas flow as the comet ages (Rickman et al., 1990). The timescale for mantle growth presumably varies inversely with the nucleus radius, but the functional form is not well established.

Equations (1), (2), (4) and (5) are plotted as functions of nucleus radius in Figure 2. The median dynamical lifetime of the SPCs, $\tau_{dyn} = 4 \times 10^5$ yr (Levison and Duncan, 1997) is also marked, as is the mean orbital period of SPCs, $\tau_{SPC} = 10$ yr. We emphasize that each of the timescales plotted in Figure 2 is uncertain, some by more than an order of magnitude, due to the unknown physical parameters of the nucleus. Nevertheless, the inequalities in the figure are so strong (the vertical axis spans 14 powers of ten) that important inferences about the rotational character of comets may still be made.

First, $\tau_{dyn} > \tau_{dv}$ means that SPCs outlive their supply of volatiles, with the consequence that many "dead" comets should exist (unless the nuclei completely disintegrate). The fact that relatively few such objects are known is presumably a consequence of observational selection against finding small, dark, defunct cometary nuclei (Luu, 1994; McFadden, 1994; Jewitt, 1996). As our surveys of the inner solar system become more complete, we should *expect* to find numerous dead and dormant cometary nuclei.

Second, $\tau_{dyn} > \tau_{ex}$ and $\tau_{dv} > \tau_{ex}$ together mean that there is ample time, and more than enough volatiles, for outgassing torques to produce excited rotational states in the nuclei of SPCs. Indeed, we should expect to find these nuclei in excited



Figure 2. The timescales for rotational spin-up, damping and devolatilization (τ_{ex} , τ_{damp} and τ_{dv} , respectively) are plotted as a function of the nucleus radius, assuming nucleus density = 1000 kg m⁻³, rotation period P = 10 h, dimensionless moment arm $k_T = 0.05$. The shaded region shows comets for which the devolatilization timescale is comparable to, or less than, the orbital period of an SPC. Comets in this region will appear as inert asteroidal bodies. Filled circles mark the rotation periods of nuclei (Table I), H = P/Halley, Hy = C/Hyakutake, HB = C/Hale–Bopp.

rotational states. The case for long-period comets (which spend a comparatively tiny fraction of each orbit in the vicinity of the sun), is less clear.

Third, $\tau_{\text{damp}} > \tau_{\text{ex}}$ for all but the largest nuclei ($r_n > 100 \text{ km}$), meaning that frictional dissipation of energy in typical cometary nuclei (all of which have $r_n < 100 \text{ km}$) is unimportant.

Fourth, $\tau_{dv} < \tau_{SPC}$ for $r_n < 0.1$ km, meaning that small comets should be rapidly devolatilized, leading to a size distribution of SPC nuclei that is truncated at small sizes. This is in qualitative agreement with the empirical under-abundance of sub-kilometer comets. Loss of volatiles from the larger comets is known to be retarded by the growth of an inert surface mantle. On the low-gravity, sub-kilometer nuclei considered here, mantle growth is probably unimportant and the loss of volatiles is assured.

Fifth, $\tau_{ex} < \tau_c$ for $r_n < 50$ m. Rotational bursting is probably an important end state for the Kreutz sungrazers, many of which are believed to be this small (MacQueen and St. Cyr, 1991). Rotational bursting may also quickly destroy the smaller fragments released from split comets.

A few mechanisms in addition to outgassing have been suggested for modifying the nuclear spin and should be mentioned here. Watanabe (1992) noted that densification and shrinkage of the nucleus (e.g., in response to thermally induced crystallization, cf. Yabushita, 1993) lead to an increase in the spin rate. Wallis (1984) noted that sub-orbital debris ejected from the poles and landing on the equator would constitute an "angular momentum drain", tending to slow the nucleus. Statistically, cometary nuclei split about once per century per nucleus (Chen and Jewitt, 1994), often when far removed from the sun and planets (Whipple and Stefanik, 1966). Outgassing acceleration of the spin above the critical frequency (Equation (1)) constitutes a plausible mechanism (Whipple, 1961). In such splitting, secondary nuclei carry away mass and angular momentum, and may leave the primary in an excited rotational state.

2.1. REAL COMETS: EVIDENCE FOR ROTATION

In principle, photometry of bare nuclei can provide the rotation period, the axis ratio of the nucleus projected into the plane of the sky, and the product of the geometric albedo with the cross section of the nucleus (Jewitt, 1991). When extended to the thermal infrared, time-resolved nucleus photometry also provides the albedo (A'Hearn et al., 1989; Campins et al., 1987; Millis et al., 1988). The method is best applied to comets that are either intrinsically weakly active when near the sun (these tend to be periodic comets) or to comets which are inactive by virtue of being far from the sun (in which case they are likely to be faint and hard to observe). The high angular resolution afforded by the Hubble Space Telescope (HST) has permitted some active comets to be studied in this way (e.g., Lamy and Toth, 1996) but with HST it is difficult to obtain enough coverage of rotational phase to define the lightcurve with confidence.

In some very active comets, aperture photometry of the coma has been used to search for periodicities caused by rotational modulation of the outgassing rate (e.g., Millis and Schleicher, 1986; McDavid and Boice, 1995; Rodriguez et al., 1997). The method is insensitive to rotational variations on timescales short compared to the crossing time for the photometry aperture employed (Jewitt, 1991) but otherwise provides a good measure of the temporal variability of the outgassing rate. However, it is still not entirely clear how the outgassing rate variability relates to the nucleus rotation. Meaningful interpretation of time-resolved coma photometry depends on prior knowledge of the number of active vents.

Some cometary comae display waves in the surface brightness that can be used to constrain the rotation period (cf. Figure 3; Sekanina, 1981b; Belton et al., 1991; Samarasinha and A'Hearn, 1991; Boehnhardt and Birkle, 1994; Cochran and Trout, 1994). In an early guise, this was known as the "halo method" (halos are successive waves of ejecta tightly coiled by nuclear rotation). The halo method frequently failed, partly because the outflow speed was not well constrained by the limited available data and partly because the temporal coverage was often



Figure 3. Waves ("halos") in the coma caused by rotational modulation of the mass loss rate. This is C/Hale–Bopp on 1997 March 3, imaged in *R* band (mostly continuum) with the 1.2-m Mt. Hopkins telescope. Field of view is roughly 200×300 arcsec. The image has been unsharp masked to enhance the periodic features. Courtesy J. Luu.

inadequate (Whipple, 1982; Sekanina, 1981a). Only recently have observations of the extended (10⁶ km-scale) gas comae of bright comets been convincingly tied to the rotation of the nucleus (Schulz und Schlosser, 1989; Cochran and Trout, 1994; Combi et al., 1994; Schulz et al., 1994).

Observational properties of relatively well-observed cometary nuclei are summarized in Table I. The main features of note include the low surface albedos (generally interpreted as evidence for carbon-rich, refractory mantles), the small fractional active areas (again indicative of widespread mantles), the elongated shapes and radii mostly in the range from a few to 10 km. Several of these properties are qualitatively understood in terms of the timescales discussed above. For example, the lack of sub-kilometer nuclei is presumably a consequence of the short τ_{dv} of these bodies. The rotation periods of most nuclei cluster near the centripetal limits (Figure 2), suggesting the action of torques on the nuclear spin. It is possible that rotational deformation (Weidenschilling, 1981) of nuclei that have been torqued towards the critical period may be partly responsible for the elongated shapes and ~6 hr minimum rotation periods in Table I.

Nucleus	P (hr) ^a	$r_e (\mathrm{km})^{\mathrm{b}}$	p_V^{c}	a/b^{d}	f^{e}	Reference
Arend-Rigaux	13.56 ± 0.16	5	0.03	1.6/1	0.1–1	1–6
P/Neujmin 1	12.67 ± 0.05	10	0.03	1.6/1	0.1 - 1	3, 7, 8
P/Encke	15.08 ± 0.08	3.5	0.04^{f}	3.5/1	0.2	9, 10
P/Halley	2.2d, 7.2d	5	0.04	2/1	10	11-15
P/Tempel 2	8.95 ± 0.01	5	0.021	1.9/1	0.1 - 1	16–21
P/SW2	5.58 ± 0.03	<3.1	0.04^{f}	1.6/1	?	22
P/Levy 1991XI	8.34	5.8	0.04^{f}	1.3/1	?	23
P/Faye	_	2.7	0.04^{f}	_	3	24
P/Swift-Tuttle	67.5 ± 0.4	11.8	_	_	3	25-31
C/Hyakutake	6.23 ± 0.03	2.1 ± 0.4	0.04^{f}	_	60	32–35
C/Hale-Bopp	11.30 ± 0.02	25	_	_	20	36–38
P/Wilson-Harr	6.1 ± 0.05	_		1.2/1	_	39

TABLE I Properties of Nuclei (adapted from Jewitt, 1991, 1996)

^a Nuclear rotation period.

^b Effective circular radius.

^c Cisual geometric albedo.

^d Projected axis ratio.

^e Active fraction \times 100.

^f Albedo assumed.

1 = Jewitt and Meech 1985; 2 = Millis et al. (1988); 3 = Birkett et al. (1987); 4 = Brooke and Knacke (1986); 5 = Tokunaga and Hanner (1985); 6 = Veeder et al. (1987); 7 = Jewitt and Meech 1988; 8 = Campins et al. (1987); 9 = Jewitt and Meech (1987); 10 = Luu and Jewitt (1990); 11 = Millis and Schleicher (1986); 12 = Sagdeev et al. (1989); 13 = Watanabe (1989); 14 = Samarasinha and A'Hearn (1991); 15 = Belton et al. (1991); 16 = A'Hearn et al. (1989); 17 = Jewitt and Luu (1989); 18 = Boehnhardt et al. (1990); 19 = Wisniewski (1990); 20 = Sekanina (1991); 21 = Mueller and Ferrin (1996); 22 = Luu and Jewitt (1992); 23 = Fitzsimmons and Williams (1994); 24 = Lamy et al. (1996); 25 = Sekanina (1981a); 26 = Yoshida et al. (1993); 27 = Boehnhardt and Birkle (1994); 28 = Jorda et al. (1994); 29 = Fomenkova et al. (1995); 30 = O'Ceallaigh et al. (1995); 31 = McDavid and Boice (1995); 32 = Larson et al. (1996); 33 = Schleicher et al. (1998); 34 = Sarmecanic et al. (1997b); 35 = Jewitt and Matthews (1997); 36 = Sekanina (1997); 37 = Rodriguez et al. (1997); 38 = Serra et al. (1998); 39 = Osip et al. (1995).

2.2. EVIDENCE FOR EXCITED ROTATIONAL STATES AND NUCLEAR SPIN-UP

The best-observed short-period comets (P/Tempel 2, P/Encke, P/Neujmin 1, P/Arend-Rigaux) have nucleus lightcurves that are singly-periodic and therefore consistent with principal axis rotation (see references in Table I). Photometric evidence for more complex rotation is occasionally claimed, e.g., for P/Tempel 2 by Mueller and Ferrin (1996), for P/Schwassmann-Wachmann 1 by Meech et al. (1993). Unfortunately, secular variations in the outgassing pattern, when combined with the typically incomplete coverage of rotational phase, may masquerade

as multiple periodicities in the lightcurve. As a result, it is surprisingly difficult to obtain compelling evidence for anything other than principal axis rotation in comets. The most convincing and famous exception is P/Halley, for which periods derived from jet curvature disagree with periods derived photometrically (Peale and Lissauer, 1989; Sagdeev et al., 1989; Watanabe, 1989; Belton et al., 1991; Samarasinha and A'Hearn, 1991), and for which in-situ images of the nucleus from spacecraft provide important additional constraints.

The rotational properties of P/Tempel 2 were established from a long series of optical measurements taken at the 1988 apparition (Jewitt and Luu, 1989; Boehnhardt et al., 1990; Wisniewski, 1990). Sekanina (1991) synthesized these measurements to obtain rotational period $P_{\text{sek}} = 8.93120 \pm 0.00006$ hr. The period was redetermined in the following orbit by Mueller and Ferrin (1996), who obtained a set of aliassed periods, the closest being $P_{1994} = 8.9392 \pm 0.0028$ hr. Their re-analysis of the 1988 data gave $P_{1988} = 8.9328 \pm 0.0031$ hr. The difference $P_{1994} - P_{1988} = 0.0064 \pm 0.0041$ hr (23 ± 15 s) is formally insignificant. However, based on the plotted lightcurves Mueller and Ferrin maintain that the difference is significant and note that, if so, it may indicate either nuclear spin-up or an un-modelled result of non-principal axis rotation.

Two independent photometric measurements of the spin period of C/Levy 1990c were obtained, with discordant results. A long series of emission line flux measurements by Schleicher et al. (1991) in late August 1990 gave $P = 18.9 \pm 0.3$ hrs. A set of IUE measurements of emission lines 3 weeks later in mid-September 1990 gave $P = 17.0 \pm 0.1$ h (Feldman et al., 1992). At the time of these observations, C/Levy was outgassing water molecules at about 1.5×10^{29} s⁻¹, corresponding to gas mass loss rate dM/dt = 4500 kg s⁻¹. Taking dimensionless moment arm $k_T = 0.05$, $\Delta \omega / \omega = \Delta P / P \sim 0.1$, outgassing velocity $V_{\text{th}} = 10^3$ m s⁻¹ and bulk density $\rho = 10^3$ kg m⁻³, we find a maximum nucleus radius $r_n = 3$ km by Equation (3). It is thus at least plausible that outgassing torques accelerated the spin of the nucleus, although specific evidence is lacking, and the different periods might instead reflect rotation in an excited state.

3. Observational Constraints on Comet Hale–Bopp

Hale–Bopp ranks as one of the best observed comets of all time, thanks to its early discovery at 7 AU heliocentric distance (Hale and Bopp, 1995). The morphology of the coma, at first driven by the sublimation of super-volatile carbon monoxide (Biver et al., 1996; Jewitt et al., 1996) and later by water (Biver et al., 1997) has been under constant scrutiny ever since discovery. The moderate semimajor axis (a = 186 AU) and eccentricity (e = 0.995) suggest that comet Hale–Bopp has made relatively few previous close approaches to the sun compared to most other comets that have been studied in detail (Bailey et al., 1996). Therefore, its nuclear



Figure 4. Left, the "starfish" morphology of C/Hale–Bopp on 1996 Nov 12 (Courtesy O. Hainaut, University of Hawaii). Right, spiral arm of 1997 Apr 5 (Courtesy W. Vacca, University of Hawaii). Field of view is 90 arcsec, North is up, East to the left. Both images were taken at the University of Hawaii 2.2-m telescope and may be viewed on the web at www.ifa.hawaii.edu/images/hale-bopp/hb_images.html.

properties may be close to pristine. From the point of view of nucleus rotation, this is probably the most significant aspect of Hale–Bopp.

Most measurements of near-nucleus coma morphology converge on a rotation period near 11.3 hours (11.30 \pm 0.02 h Farnham and Schleicher, this conference; 11.34 \pm 0.02 h Licandro et al., this conference; 11.3 \pm 0.1 h Sarmecanic et al., 1997a). The previously reported oscillation of the period between 11.2 \pm 0.1 and 11.65 \pm 0.1 h with a 22 \pm 2 day 'super-period' (Jorda et al., 1997) has been retracted (Jorda et al., this conference).

The appearance of the jets in the near-nucleus coma changed dramatically with time. Prior to February 1997, the jets were straight, near-radial, and changed little on timescales of hours and days (Figure 4a). As perihelion approached, the jets became more curved, eventually giving rise to a classical set of spiral arms clearly reflecting the rotation of the nucleus (Figure 4b). After May 1997, the "starfish" morphology gradually re-appeared (although post-perihelion optical observations were severely hampered by the small solar elongation and growing southerly declination of the comet). A simple interpretation is that the straight jets were the edges of emission cones rooted in active areas on the nucleus and traced out by the rotation (Sekanina, 1997). This neatly accounts for the invariance of the straight jets even as the nucleus rotated underneath with its ~11.3 h period. Modelling suggests that the jets will appear straight provided the rotation vector is inclined to the line of sight by $\geq 50^{\circ}$. At smaller angles, the curvature of the jets is projected into the plane of the sky, producing characteristic spiral arms. The emergence of



Figure 5. Map of the sky showing the apparent motion of Hale–Bopp in the plane of the sky. Near and after perihelion, the morphology of the comet adopted a spiral form consistent with a small angle between the viewing direction and the rotation vector. Before February 1997 and after May 1997, the comets reverted to a "starfish" morphology, consistent with a more nearly equatorial view. Solid diamonds mark some of the pole directions already reported in the literature, S = Sekanina, 1997, or at the Hale–Bopp conference: B = Biver et al.; J = Jorda et al., ML = Metchev and Luu; S-R = Serra-Ricart et al., and 3 poles suggested by Sekanina and Boehnhardt SB1, SB2, SB3. A swath of allowed positions (not plotted) has been identified by Samarasinha et al. (this conference), to whose interesting paper the reader is referred.

spiral arms in Hale–Bopp near perihelion presumably indicates that the nucleus was viewed nearly parallel to its rotation vector at this time.

The basic emission cone (or "fan") model has been used by Sekanina and Boehnhardt (this conference) to derive a pole near RA, Dec = 170° , -40° , and by Metchev and Luu (personal communication) near RA, $Dec = 290 \pm 15^{\circ}$, -42 \pm 8°. Several variations on the emission cone hypothesis have been proposed. Samarasinha incorporated diurnal modulation of the outgassing flux in proportion to the cosine of the instantaneous zenith angle of the sun as seen from each vent. He finds that some coma jets are projection effects at the edges of emission cones, as above, while others correspond to local maxima in the diurnal outgassing rate. Sekanina and Boehnhardt assumed the outgassing rate to vary rapidly with time, giving a series of short-duration (< 8 min) emission spikes that can be made to fit the jets rather well. Biver used the asymmetry of CO rotational lines to infer a pole near RA, Dec = 325° , -53° . Pole directions proposed at the Hale–Bopp Conference and elsewhere are plotted in Figure 5. The scatter in Figure 5 shows that (as of April 1998) there is no close agreement concerning the pole direction of Hale-Bopp. This provides a measure of the difficulty in determining the rotation pole even in this very well observed comet.

Is there evidence for non-principal axis rotation of the nucleus of Hale–Bopp? Perhaps the best constraint is provided by the stability of the jet position angles measured over several months in late 1996. Serra-Ricart et al. (this conference) measured a periodic oscillation in jet position angles with period 24 ± 1 day, range 20°. The oscillation is significant at the 97% (roughly 2σ) confidence level. It will be crucial to determine whether this oscillation is confirmed in independent data. Rodriguez et al. (1997) reported evidence for two periods in photometry of the coma obtained between 1997 March 4 and 20. The shorter of these (5.5 hr) is presumably half the rotation period deduced independently from jet morphology. The longer period (7.19 days) suggests a slow nodding of the nucleus with attendant modulation of the outgassing rates, but it is not clear how this period can be reconciled with the 24 day period of the jet position angles. However, the photometric range of variation is very small (0.04 mag. for the 7.19 day period and 0.02 mag. for the 5.5 hr period) and, again, independent confirmation of this photometric result would be invaluable.

The total gas mass lost by Hale–Bopp during its current apparition was $\Delta M = 6 \times 10^{12}$ kg. The mass of a 20 km radius spherical nucleus of density $\rho = 10^3$ kg m⁻³ is $M = 3.4 \times 10^{16}$ kg, giving $\Delta M/M \sim 2 \times 10^{-4}$. With $V_{eq} = r_n \omega = 4$ m s⁻¹, $V_{th} = 1000$ m s⁻¹, and $k_T = 0.05$ the maximum possible change in the angular momentum due to outgassing is $\Delta L/L = \Delta \omega/\omega = 0.2\%$ (Equation (3)), corresponding to a change in the rotation period of 1.7 min. Submillimeter measurements (by the author and Henry Matthews) indicate a dust mass loss rate roughly 5 to 10 times the gas production rate. The submillimeter dust leaves the nucleus at about 1/10th the gas speed, and so its contribution to the momentum rivals that of the gas. The potential change in the rotation period (~3 min) is comparable in size to the reported uncertainties on the rotation period (e.g., Farnham and Schleicher, this conference). Therefore, it would not be futile to search for pre-perihelion vs. post-perihelion spin-up of C/Hale–Bopp.

4. Summary

For most nuclei, the timescale for excitation of the spin by jets is shorter than the frictional damping time, and shorter than the devolatilization and dynamical lifetimes in the inner solar system. Therefore, we expect that cometary nuclei should, in general, occupy excited rotational states.

Paradoxically, evidence for excited rotational states in comets is in short supply. The best (and almost only) case is that of P/Halley. The general lack of evidence for non-principal axis rotation in comets may be an artifact of the limited quantity of data available on most comets, rather than an indication of the absence of excited motions.

The richly varying morphology of the coma of C/Hale–Bopp provides many hints concerning the magnitude and direction of the nucleus spin vector. However,

while the determination of the rotation period (11.3 h) appears secure, there is presently no consensus regarding the pole direction, and only limited evidence for multiple periodicities that might indicate an excited rotational state.

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Appendix A: Acceleration and Torque on the Nucleus

Outgassing from comets proceeds through a small number of active vents in an otherwise inactive surface crust or mantle (Rickman et al., 1990; Keller et al., 1994; Kührt and Keller, 1994). Recoil forces from outgassing can change both the linear momentum and the angular momentum of the nucleus. The vector force on the nucleus in the anti-solar direction is

$$\vec{F} = \sum_{i} \left. \frac{\mathrm{d}m}{\mathrm{d}t} \right|_{i} \vec{V}_{\mathrm{th}} \bullet \vec{s}_{i} \tag{A1}$$

where $dm/dt|_i$ (kg s⁻¹) is the mass loss rate from the i_{th} vent, \vec{V}_{th} (m s⁻¹) is the velocity of the outgassed material, \vec{s}_i is the unit vector in the direction towards the sun and the sum is taken over all vents. Likewise, the vector torque on the nucleus (cf. Figure 1) is the quantity

$$\vec{T} = \sum_{i} \left. \frac{\mathrm{d}m}{\mathrm{d}t} \right|_{i} \vec{V}_{\mathrm{th}} \times \vec{r}_{i} \tag{A2}$$

averaged over the rotation period of the nucleus, where $\vec{r_i}$ is the vector from the center of mass to the vent. An exact calculation of force and torque would require detailed knowledge of the shape of the nucleus, the distribution of active vents over its surface and the thermo-physics of sublimation (which controls the diurnal variation of dm/dt). Such a calculation is beyond our present grasp. It is nevertheless instructive to consider a simplified model of the nucleus from which we may obtain crude but useful estimates of the scalar recoil force $F = |\vec{F}|$, and torque $T = |\vec{T}|$. The model will also serve to show how an asymmetrically located jet on a symmetric nucleus gives rise to a torque.

It is convenient to write the scalar recoil force as

$$F = k_A V_{\text{th}} \, \mathrm{d}M/\mathrm{d}t \quad (0 \le k_A \le 1). \tag{A3}$$

where dM/dt is now the total mass loss rate and the dimensionless constant, k_A , is a measure of anisotropy of the mass loss, with $k_A = 1$ corresponding to perfectly



Figure A1. Idealised representation of a cometary nucleus rotating at angular rate $\omega = 2\pi/P$ (P = nucleus rotation period) and supporting a single active area (black). The moment arm of the torque is ℓ . The sun is below the nucleus in this figure: straight arrows signify mass loss due to sublimation from the active area.

collimated, uni-directional emission and $k_A = 0$ corresponding to the isotropic case. Likewise, we write the scalar torque on the nucleus as

$$T = k_T V_{\text{th}} r_n \, \mathrm{d}M/\mathrm{d}t \quad (0 \le k_T \le 1) \tag{A4}$$

where r_n (m) is the nucleus radius. The dimensionless constant, k_T , is a measure of the effective moment arm of the outgassing, with $k_T = 1$ corresponding to tangential emission and $k_T = 0$ corresponding to perfectly central outgassing. The essence of the problem is to calculate the two constants, k_A and k_T , which characterize the response of the nucleus to outgassing forces.

A schematic model nucleus, a rectangular block of dimensions $2s \times 2r$ supporting a single active vent, is illustrated in Figure A1. The mass loss vector is assumed to act perpendicular to the surface, with an impact parameter of length ℓ , and the nucleus is assumed to be illuminated equatorially. The mass loss rate is a function of the time of day. For sublimation in equilibrium with sunlight, I take

$$\frac{\mathrm{d}M}{\mathrm{d}t}(t) = \frac{\mathrm{d}M}{\mathrm{d}t}(0)\cos\left(\frac{2\pi}{P}t\right)$$

for $-P/4 \le t \le P/4$ and dM/dt(t) = 0 otherwise (i.e., I assume that night-side emission is zero, cf. Figure A1).

By Equation (A1), the recoil force on the nucleus, averaged over a rotation period, is given by

$$\overline{F} = \frac{\int_{-P/4}^{P/4} V_{\rm th} \frac{\mathrm{d}M}{\mathrm{d}t}(0) \cos^2\left(\frac{2\pi}{P}t\right) \mathrm{d}t}{P}.$$

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which simplifies to

$$\overline{F} = \frac{1}{4} V_{\rm th} \frac{\mathrm{d}M}{\mathrm{d}t}(0)$$

Comparing with Equation (A3) we see $k_A = 1/4$. The same block model illuminated parallel to the rotation vector gives $k_A = 1$. Calculations for nuclei with more realistic, prolate-spheroid, shapes also give $k_A \sim$ few tenths, so that the rectangular block model described here is at least qualitatively valid. Marsden (this conference) has measured the non-gravitational acceleration of C/Hale-Bopp as $A_1 = (1.04 \pm 0.03) \times 10^{-8} \text{ AU day}^{-2} (2 \times 10^{-7} \text{ m s}^{-2}).$ We put $F = M A_1$ and substitute $V_{\text{th}} \sim 10^3 \text{ m s}^{-1}$ (Biver et al., 1997), $dM/dt(0) = 3 \times 10^5 \text{ kg s}^{-1}$ (Despois et al., this conference) and $0.25 \le k_A \le 1$ to find $0.4 \times 10^{15} \le M \le 1.5 \times 10^{15}$ kg. This is the mass of an ice sphere of diameter $10 \le D \le 14$ km, about a factor of 3 to 4 smaller than the best guess diameter of C/Hale–Bopp (Wink et al., this conference). The uncertainties in the calculation (e.g., is the density less than 10^3 kg m⁻³? Is A1 accurately measured?) are such that we see no immediate cause for concern about the disagreement between the recoil mass and the nucleus size inferred from optical/IR data. As the parameters of the spin state of the nucleus and of the nongravitational acceleration become better defined, we expect to be able to make a more exact calculation of the recoil mass.

By Equation (A2) the rotationally averaged scalar torque is

$$\overline{T} = \frac{\int_{-P/4}^{P/4} \ell V_{\rm th} \frac{\mathrm{d}M}{\mathrm{d}t}(0) \cos\left(\frac{2\pi}{P}t\right) \,\mathrm{d}t}{P}$$

which simplifies to

$$\overline{T} = \frac{\ell V_{\rm th}}{\pi} \frac{\mathrm{d}M}{\mathrm{d}t}(0).$$

The active area could appear anywhere along the side of the nucleus, so

$$|\overline{\ell}| = r/2, \overline{T} = \frac{rV_{\text{th}}}{2\pi} \frac{\mathrm{d}M}{\mathrm{d}t}(0),$$

and, by Equation (A4), $k_T = (2\pi)^{-1} \sim 0.16$.

I have calculated torques and dimensionless moment arms, k_T , for model nuclei of other shapes, always under the assumptions that the outgassing vector is normal to the local surface and that sublimation proceeds from a small number of active areas distributed at random. For single active areas on a nucleus with equatorial axis ratios ~ 2:1 (as suggested by the nucleus of P/Halley, Keller et al., 1987) and by rotational lightcurves (Table I), I obtain values $k_T \sim 0.1$, similar to the value obtained from the crude block nucleus model. For N jets distributed randomly in

azimuth, $k_T \propto N^{-1/2}$. Observations suggest that the number of strongly active vents is small (Sekanina, 1990; Figure 4), perhaps $N \sim 4$. Accordingly, I take $k_T = 0.05$ as the best estimate of the dimensionless moment arm. Note that little is known about the physics of vent formation (Rickman et al., 1990; Kührt and Keller, 1994) and thus we cannot be sure that the active areas are randomly distributed around the nucleus, as this last step assumes.

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