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Three-Dimensional Reconstruction of Asteroids from Radar Observations

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ABSTRACT

A program of research has recently been initiated under the NASA Innovative Research Program to develop and test techniques for reconstructing the three-dimensional shapes of asteroids from ground-based radar observations. Presently, no methods exist to do this and the full scientific potential of current radar data sets lies untapped. The situation will become even more pronounced in approximately two years when the upgraded Arecibo radar system becomes capable of regularly producing asteroid data sets with very fine spatial resolution and high signal-to-noise ratios. If successful, the proposed research will significantly increase our knowledge of these objects. The potential and limitations of the techniques developed will be rigorously tested in a series of computer simulations and optical scattering experiments with scale models.

INTRODUCTION

Over 5,000 asteroids are currently known, and new ones are rapidly being discovered, yet the asteroid population is as mysterious as it is abundant (Binzel, 1989). Galileo's images of 951 Gaspra have provided our first, and to date only, close-up glimpse of one of these strange worlds (Belton et al., 1992). Asteroids most likely hold important keys to understanding the origin and evolution of the solar system, and judging from meteorite samples and ground-based observations, they are a store of tremendous mineral wealth. With some near-Earth asteroids (NEAs) being more energetically accessible than the Moon, these bodies will one day be important as subjects for solar system exploration and exploitation. The fact that NEAs periodically impact the Earth, with consequences possibly as extreme as global catastrophe, underscores the importance of knowing as much as possible about these objects.

As the most distinguishing characteristic of a small body is its three-dimensional shape, we cannot begin to fully understand the astroid population unless we have the capability to determine the shapes of large numbers of them from remotely sensed data. In addition, the accuracies with which several other asteroid properties can be estimated are limited by the extent to which the shape is known. Examples are: optical and radar albedos, which provide important constraints on asteroid composition with radar albedo being a particularly effective way to identify metal-rich asteroids (Ostro et al., 1991); the position of an asteroid's center of mass (COM) with respect to radar ranging and/or Doppler data, knowledge of which can result in highly refined orbits (Yeomans et al., 1992); and determination of asteroid pole directions from optical and radar data. Indeed, the interpretation of almost all asteroid observational data is affected to some extent by our knowledge of the object's shape.

At the present time, three-dimensional models of asteroids are limited to simple, symmetric, convex objects such as spheres, ellipsoids, and spherically-capped cylinders. In fact, there has not previously existed the motivation to develop more complex models because no observational technique could provide the resolution required to allow detailed constraints to be placed on three-dimensional shape. Recent radar observations, however, have shown that some asteroids are highly irregular, possibly nonconvex bodies that cannot be accurately described by such simple models (Ostro et al., 1990; Ostro et al., 1991).

Of the many ways to observe and study asteroids, radar is one of the most powerful. It is the most effective ground-based technique for spatially resolving an asteroid, and hence the most powerful for providing detailed information about three-dimensional shape. Radar's importance, already very significant, will grow tremendously in less than two years with the completion of the NSF/NASA upgrade to the Arecibo telescope that will increase the radar sensitivity of that instrument by an average factor of approximately 15. Also, the Golstone radar telescope, which is part of NASA's Deep Space Network, has twice the sky coverage of Arecibo and enough sensitivity to produce high-resolution images of Earth-approaching asteroids and comets. Given the improvements in both radar systems during the next few years and the increasing rate of discovery of Earth-approaching objects, one can expect increasingly frequent imaging opportunities. Therefore, in the near future high signal-to-noise-ratio asteroid delay-Doppler distributions with spatial resolutions as fine as 30 meters should become commonplace, and for the first time the potential will exist to study in detail the three-dimensional shapes of a large number of asteroids—provided reliable techniques to do so are available.

With this backdrop as motivation and with support from a NASA Innovative Research Program grant, I have recently begun a program of research designed to develop and test techniques for three-dimensional asteroid shape reconstruction from radar data. This project will involve myself and a Ph.D. student working closely together for a two year period. A large component of the research involves the construction and use of a laboratory laser radar system for making optical delay-Doppler measurements on scale asteroid models. Measurements made with this system will be used in testing the shape reconstruction techniques that we develop.

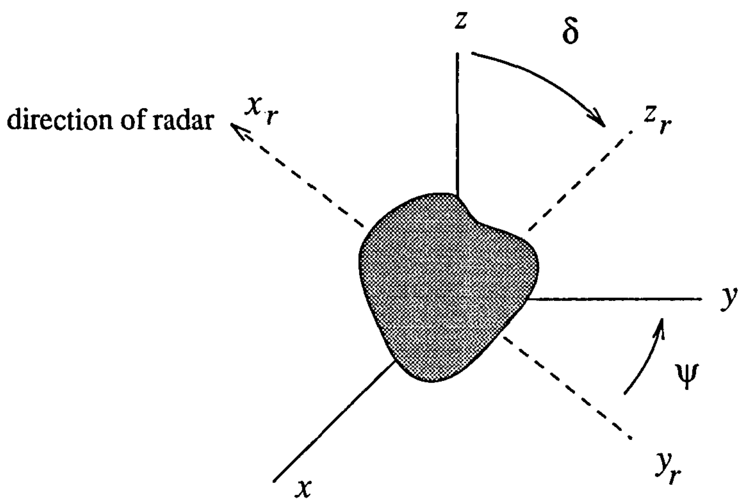

FIGURE 1 Asteroid and radar coordinates. The x,y,z system, "asteroid coordinates," is fixed on the asteroid with the z axis coinciding with the pole direction and the x and y axes defining the equatorial plane. The x_r, y_r, z_r system, "radar coordinates," is fixed with respect to the radar such that the x_r axis points towards the radar and the y_r axis lies in the equatorial plane. The relation between these coordinate systems is specified by two angles. The angle between the z and z_r axes is called the *subradar latitude*, and is denoted by δ; it is essentially constant during a radar observation. The angle between the y and y_r axes is called the *rotational phase*, and is denoted by ψ; it increases linearly with time as the astroid rotates.

DESCRIPTION OF RESEARCH

Delay-Doppler-Resolved Radar Data

Planetary radars measure backscattered power, which in normalized form is called radar cross section and is denoted by the symbol σ. With appropriate techniques, σ can be resolved in time delay $\tau = 2R/c$ with R the distance between the radar and a point on the asteroid and c the speed of light, and Doppler frequency $\nu = -(2/\lambda)(dR/dt)$ with λ the radar wavelength (Ostro 1989). The corresponding spatial resolution can be understood with reference to the coordinate systems described in Figure 1. The relation between these systems is

$$x_r = (x\cos\psi - y\sin\psi)\cos\delta + z\sin\delta \tag{1}$$

$$y_r = x\sin\psi + y\cos\psi \tag{2}$$

$$z_r = -(x\cos\psi - y\sin\psi)\sin\delta + z\cos\delta. \tag{3}$$

It can be shown that, relative to the COM, the time delay of a point is proportional to its x_r coordinate, and its Doppler frequency is proportional to its y_r coordinate. Thus the delay-Doppler distribution of radar cross section, $\sigma(\tau,\nu;\psi,\delta)$, resolves the asteroid along the orthogonal directions x_r and y_r. Generally there are two or more points having given delay and Doppler values (Figure 2), and

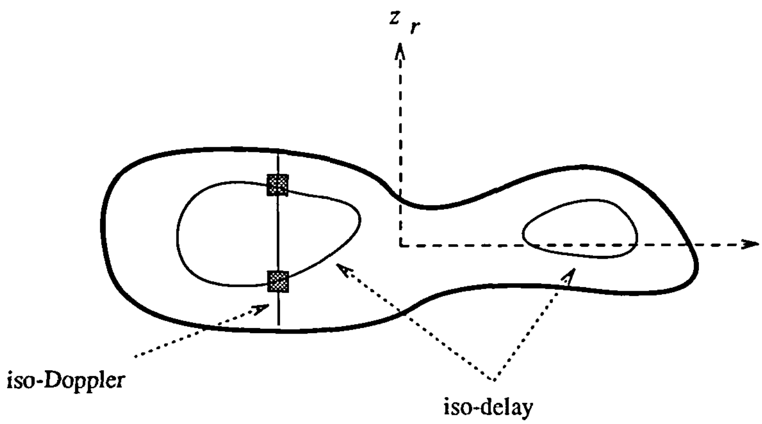

FIGURE 2 Resolution of an asteroid in delay and Doppler. The illustration shows an asteroid as seen from the radar's perspective, far away on the x_r axis. The z_r axis corresponds to the projection of the asteroid's pole onto the plane of the sky. Iso-Doppler contours are curves of constant y_r. Iso-delay contours are curves of constant x_r. The gray squares show two points having the same values of delay and Doppler.

the corresponding resolution cell of the delay-Doppler distribution is the sum of the radar cross sections of all of them. The many-to-one aspect of this mapping is referred to as the *North-South ambiguity*.

Central to the physical interpretation of radar backscatter is the concept of an *angular scattering law*, $\sigma_0(i)$. This describes the dependence of the radar cross section of a surface patch on the angle of incidence. We write

$$d\sigma = \sigma_0(i)\,dS \tag{4}$$

where $d\sigma$ is the radar cross section of a surface patch of area dS when the angle of incidence is i. The angular scattering law is a function of the small-scale structure, or roughness, or the surface. The scattering-law model most commonly used in asteroid radar astronomy has the form

$$\sigma_0(i) = \rho\cos^n i \tag{5}$$

where ρ is a measure of the reflectivity of the surface material and n describes how specular the scattering is, e.g., for very large n, σ_0 is essentially zero except for normal incidence, $i = 0$.

Modeling of Shape

As for large-scale surface structure, or shape, a simply connected body can be thought of as a deformed sphere. As a result there exists a one-to-one mapping between points on the body, $\mathbf{r} = (x, y, z)$, and points on a sphere, indexed by the spherical angles θ, ϕ. The body can therefore be described by the equation

$$\mathbf{r} = \mathbf{r}(\theta, \phi). \tag{6}$$

Denoting the spherical coordinates of $\mathbf{r}$ by r, θ_a, ϕ_a, this can be expressed as the scalar equations

$$r = r(\theta,\phi) \tag{7}$$

$$\theta_a = \theta_a(\theta,\phi) \tag{8}$$

$$\phi_a = \phi_a(\theta,\phi). \tag{9}$$

These equations are much simplified by the assumption $\theta_a = \theta$, $\phi_a = \phi$, which implies that a ray in any direction from the COM intersects the surface at one, and only one, point. The body is then completely specified by the single scalar function $r(\theta,\phi)$. Models of this form are probably sufficiently general to account for the great majority of physically plausible asteroid shapes.

For computational purposes, the function $r(\theta,\phi)$ needs to be parameterized in some fashion. An appropriate way to do this is with a spherical harmonic series:

$$r(\theta,\phi) = \sum_{l=0}^{N} \left[a_{l0} P_l^0(\cos\theta) + \sum_{m=1}^{l} (a_{lm}\cos(m\phi) + b_{lm}\sin(m\phi)) P_l^m(\cos\theta) \right]. \tag{10}$$

In this manner the surface is described by the $(N+1)^2$ coefficients a_{lm}, b_{lm}. As N, the order of the series, grows, finer surface features can be represented, but in practice the number of coefficients will be limited by the number of observed data.

Reconstructing Shape from Delay-Doppler Data

For a given asteroid the goal is to compute the appropriate values of the $(N+1)^2$ shape parameters a_{lm}, b_{lm} using delay-Doppler distributions observed at various rotational phases. Often we will also need to solve for the scattering law and the subradar latitude δ. This constitutes an inverse scattering problem: Given radar observations $\sigma(\tau,\nu;\psi,\delta)$, calculate a_{lm}, b_{lm}, and possibly $\sigma_0(i)$, and δ.

Before we attack the inverse problem we want to have a solution to the forward scattering problem: Given a_{lm}, b_{lm}, $\sigma_0(i)$, and δ, calculate $\sigma(\tau,\nu;\psi,\delta)$. The formal solution is

$$\sigma(\tau,\nu;\psi,\delta) = \iint h_\tau[\tau - \tau(\mathbf{r};\psi,\delta)] h_\nu[\nu - \nu(\mathbf{r};\psi,\delta)] v(\mathbf{r};\psi,\delta) \sigma_0(i(\mathbf{r};\psi,\delta))\, dS, \tag{11}$$

where the integration is over the asteroid surface. Here $h_\tau[\tau]$ and $h_\nu[\nu]$ are the radar system impulse response functions in delay and Doppler, $\tau(\mathbf{r};\psi,\delta)$ and $\nu(\mathbf{r};\psi,\delta)$ are the delay and Doppler values of the point $\mathbf{r}$ at the given rotational phase and subradar latitude, and $v(\mathbf{r};\psi,\delta)$ is the surface visibility function ($v = 1$ if the point $\mathbf{r}$ is visible to the radar, $v = 0$ if it is shadowed). Eq. (10) enters into (11) implicitly since it determines the coordinates and surface normals of points on the surface and hence defines the functions $\tau(\mathbf{r};\psi,\delta)$, $\nu(\mathbf{r};\psi,\delta)$, $i(\mathbf{r};\psi,\delta)$, and $v(\mathbf{r};\psi,\delta)$.

Eq. (11) is the starting point for attacking the inverse problem and reconstructing an asteroid's shape. Ideally we would be able to invert it analytically, or at least, say, derive from it a differential equation for $r(\theta,\phi)$ which could then be used to show that under appropriate conditions there exists a unique solution. Results of this kind have been obtained for related problems in the field of machine vision (Horn and Brooks, 1989) such as stereo vision, shape-from-shading, and photometric stereo. However, the present problem differs from these related ones in significant ways. The largest is the presence of the North-South ambiguity. These machine vision techniques utilize images with a one-to-one correspondence between position on the object and position in the image, something delay-Doppler distributions for a small body do not provide. Also, these other techniques do not generally consider shadowing effects as these have proven very difficult to deal with. Yet shadowing plays a very important role in determining delay-Doppler distributions (through the visibility function $v(\mathbf{r};\psi,\delta)$). These factors make it unlikely that the asteroid-shape-from-radar problem will prove to be analytically tractable. Hence, while we are devoting a small portion of our time to investigating possible analytical solutions, most of our efforts are directed towards numerical solutions.

Numerically, a reconstruction can be treated as a chi-square (χ^2) minimization problem. For any candidate shape we can compute the corresponding delay-Doppler distribution by evaluating (11). The delay-Doppler residuals are

$$\epsilon_{\tau\nu\psi} = \sigma_{\rm obs}(\tau,\nu;\psi,\delta) - \sigma(\tau,\nu;\psi,\delta), \tag{12}$$

where $\sigma_{\rm obs}$ refers to the observed radar data and σ to the modeled data. Using these we can evaluate

$$\chi^2 = \sum_{\psi}\sum_{\tau}\sum_{\nu}\left(\frac{\epsilon_{\tau\nu\psi}}{s_{\tau\nu\psi}}\right)^2, \tag{13}$$

where $s_{\tau\nu\psi}$ denotes the standard deviation of the noise fluctuations for the given residual, and this gives us a measure of the goodness of fit. Shape reconstruction is then the process of minimizing χ^2 with respect to the $(N+1)^2$ shape parameters a_{lm}, b_{lm}. If the scattering law and subradar latitude are unknown, they also need to be treated as free parameters in the fit.

Although we do not need to obtain a reconstruction in real time, we do need one in a reasonable amount of time, yet computing the integral in (11) is not trivial. Since the number of function evaluations required to minimize χ^2 grows with the number of shape parameters, in practice the number of shape parameters that can be solved for will necessarily be limited by how fast (11) can be evaluated. Accordingly, most of our initial research has been exploring the most efficient ways to code (11) and discovering which numerical minimization techniques are most appropriate to this problem. This will have a significant impact on the other components of the research because the faster we can converge on solutions, the more simulations and model reconstructions we can perform and the more extensively we can test the technique.

Testing Reconstruction Effectiveness

Although we can always find a minimum of χ^2, before we accept the resulting shape we need to have confidence that this technique is likely to produce a good reconstruction. The most direct and satisfying way to establish the technique's effectiveness would be to compare its results with "ground truth" (e.g., images from a close-encounter space mission) for a wide variety of test cases. Unfortunately, no ground truth currently exists for any radar-detected asteroid, and it may be decades before enough does exist to form a statistically meaningful test set. In the mean time we must rely on less direct tests.

These tests need to address the issues of uniqueness, stability, and sensitivity to the scattering law model. Concerns about uniqueness center on the possibility that two or more distinct shapes might give equally good fits to the observed data. Because χ^2 is a nonlinear function of the shape parameters there can exist more than one minimum, and if these minima result in similar fits to the data then the asteroid reconstruction is not unique. We anticipate that uniqueness will not prove to be a problem. Delay-Doppler distributions at several rotational phase values should always be available, and, provided $\delta \neq 0$, the sequence of x_r and y_r values, hence delay and Doppler values, that a point (x, y, z) takes on as ψ varies is unique to that point. This is evident from Eqs. (1) and (2); as functions of ψ, x_r and y_r uniquely specify the point (x, y, z), provided $\delta \neq 0$. Hence it is unlikely that two quite different asteroid models could account for the same set of delay-Doppler distributions. This is particularly true if, as will often be the case in the future, radar observations will be available for two or more distinct dates and hence probably at least two distinct values of δ. For near-Earth asteroids this due to the fact that they can move through the sky many tens of degrees while they are close enough for radar imaging. For mainbelt asteroids this would result from observations taken at two or more oppositions.

One way we will address the uniqueness question, for a given data set, is by performing several χ^2 minimizations starting from a variety of different initial conditions and seeing whether they converge to the same result or not. The uniqueness issue will also be addressed by performing reconstructions of many different simulated or laboratory model objects. If uniqueness is a problem then many of these reconstructions will fail by getting stuck in "incorrect" minima.

Stability refers to the possibility that the reconstructed model is not well constrained by the data, and small changes in the data will produce large changes in the model. Since all real data are corrupted by noise this would mean that the reconstruction would be questionable. Fortunately, this is easy to test using Monte Carlo methods. The best-fit delay-Doppler distributions will be used to generate synthetic data sets having the same noise statistics as the observed data. By comparing the shapes reconstructed from many of these synthetic data sets we will be able to place quantitative bounds on the uncertainty in the shapes due to noise.

Uniqueness and stability can be studied through computer simulation since they are issues relating to the computer models used in the reconstruction process. However, an issue that is more difficult to deal convincingly with computer simulation is that of the sensitivity of a reconstruction to scattering law model accuracy. In order to evaluate (11), we must use some analytical/numerical model for $\sigma_0(i)$, and *any* model for a process as complex as rough surface scattering will necessarily be an idealization, potentially glossing over important aspects of the phenomenon. The concern is that if reconstructions are very sensitive to changes in $\sigma_0(i)$, then any given $\sigma_0(i)$ model might introduce systematic errors into the reconstructed shape. Although some probing in this area could be done through computer simulations using a variety of models for $\sigma_0(i)$, such exercises can degenerate into demonstrations that one model can or cannot account for another model, without ever saying anything about the real world.

A more satisfying way to treat this issue is afforded by the fact that the scattering of light by rocks or clay models is in many ways analogous to the scattering of radio waves by an asteroid. At a radar wavelength of 13 cm a 2 km asteroid is 15,000 wavelengths across; at an optical wavelength of 632 nm a 1 cm rock is also 15,000 wavelengths across. Each has a surface characterized by both large-scale structure or shape, and random small-scale structure or roughness. Because of this analogy, if a modeling technique can successfully reconstruct laboratory models from optical delay-Doppler data, we can have confidence that the same technique will produce accurate reconstructions of asteroids from analogous radar data.

A system for recording optical delay-Doppler distribution is diagrammed in Figure 3. It is a slightly modified Michelson interferometer (Goodman, 1985). The light source emits a wavefront having a coherence length l_c (about 0.5 mm for the source that we are using). The beam splitter separates this into two wavefronts which follow the dashed and dotted paths. One is scattered by the model while the other is reflected by the mirror, and the beam splitter causes both to subsequently fall onto the CCD array. The lens in the dashed path results in the model being imaged onto the array while the absence of a lens in the dotted path produces a uniform plane wave at the array. The image of the model on the CCD array would appear similar to Figure 2.

The y_r coordinate of a CCD pixel determines which iso-Doppler contour it corresponds to. Its iso-delay contour is determined by moving the mirror so that h varies slowly with time. If h_0 is the distance from the beam splitter to a point on the model, then when h is within approximately $l_c/2$ of h_0 the pixel on which this point is imaged will fluctuate due to coherent interference between the two wavefronts. For other values of h the wavefronts are not coherent and there will be no fluctuations. By recording a sequence of such images with a video digitizer and noting when a particular pixel fluctuates, it can be determined which iso-delay contour it lies on. In this manner the optical delay-Doppler distribution of the model can be calculated, and by placing the model on a dual-axis mount, the subradar latitude and rotational phase, δ and ψ, can be varied as desired.

Using this system, optical data sets analogous to current and possible future radar data sets will be recorded for a wide variety of physical models, and reconstructions will be performed and compared to the actual objects. By examining the results for many different shapes and surface characteristics, all the accuracy

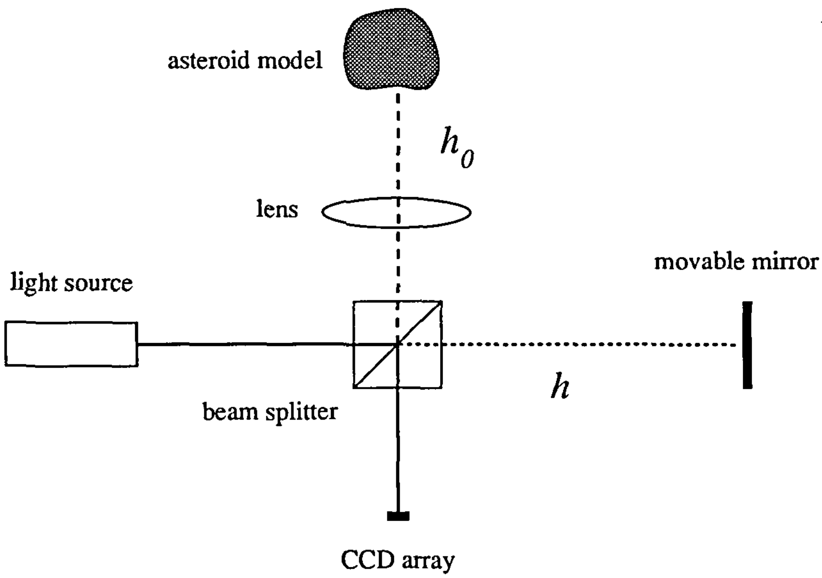

FIGURE 3 System for recording optical "delay-Doppler" distributions. The asteroid model is imaged on the CCD array, and the location of a point's image on the array determines the y_r coordinate of that point. By moving the mirror and noting the value of h for which its image fluctuates, the point's x_r coordinate can be determined.

issues discussed above will be thoroughly tested, and the potential and limitations of asteroid reconstruction from radar data will be established.

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