Fractal clustering of metamorphic veins
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ABSTRACT
Metamorphic veins record the fracture-controlled flow of fluids throughout the oceanic and continental crust. I show that the spatial distributions of veins from three diverse metamorphic settings are fractal and self-similar. Vein densities were measured by counting the number of macroscopic veins intersected along linear transects. The localities included wollastonite-quartz veins in marbles, for which the fractal dimension $D$ is 0.46, actinolite-chlorite veins in hydrothermally altered oceanic diabases ($D = 0.81$), and epidote-quartz veins in contact-metamorphosed basalts ($D = 0.25-0.63$). The fractal clustering of veins provides a geometric framework for understanding spatial and temporal patterns of fluid flow and mineral reaction during metamorphism.

INTRODUCTION
By transporting mass and helping to control the rates and extents of mineral-fluid reactions, $H_2O-CO_2-NaCl$ fluids play an important role in metamorphism. The extent to which the fluid phase participates in metamorphism hinges in part on its spatial distribution in the rock matrix. Metamorphic fluids migrate through fractures that range from microscopic arrays with widths, lengths, and spacings similar in scale to constituent mineral grains, to macroscopic arrays with widths, lengths, and spacings on the order of tens of centimetres to metres. In a typical metamorphic event, the scale and abundance of fractures changes in time and space as a complex response to changes in pressure and temperature (Norris and Henley, 1976; Norton and Knight, 1977; Yarlede, 1986; Manning and Bird, 1991). Throughout the event, the fractures are filled by minerals precipitated from flowing fluids. Key elements of the history of fluid flow are thus recorded for each metamorphic event in a crosscutting sequence of mineral-filled fractures, or veins, and associated reaction zones in the wall rock. The spacing of fractures controls the degree to which fluids pervade the rock matrix or are segregated into discrete fracture-controlled flow zones (e.g., Walther and Wood, 1984). However, there has been little effort toward developing a rigorous geometric framework within which to evaluate the spatial patterns of fractures and veins in the context of metamorphism.

Most fracture arrays in natural materials are not evenly spaced. Although fracture distributions may appear purely random, it has long been known that they cluster in groups or swarms to varying degrees. Recently, many geologic fracture sets have been shown to have fractal distributions (e.g., Barton and Larsen, 1985; Chilès, 1988; La Pointe, 1988; Barton and Hsieh, 1989; Velde et al., 1990, 1991). Other aspects of fractures are also fractal: experimental studies show that the spacings and surface roughness of laboratory-induced microfractures are fractal (Brown and Scholz, 1985; Hirata et al., 1987); fractal geometries characterize fault lengths and epicenter distributions in zones of active seismicity (e.g., Kagan and Knopoff, 1980; Aviles et al., 1987; Okubo and Aki, 1987); and the erosional patterns that develop in fractured rocks result in fractal topography (Norton and Sorensen, 1989). Below I show that veins from three diverse metamorphic environments also have fractal spacings. This implies that fracture-controlled metamorphic fluid flow has spatial structure over a range of scales. Recognition of this structure aids petrologic and geochemical studies of fluid-rock interaction.

FRACTAL ANALYSIS

Methods
A fractal set has a Hausdorff-Besicovitch, or fractal, dimension $D$ greater than the topologic dimension of the objects that constitute the set (Mandelbrot, 1983; Turetta, 1992). Fractures and veins exposed in an outcrop surface can be taken as lines with topologic dimension 1 in a plane with topologic dimension 2. If this plane were completely filled by fractures, $D$ would be 2; so fractal veins or fractures on a two-dimensional surface are characterized by $1 < D < 2$ (Barton and Larsen, 1985; Chilès, 1988; La Pointe, 1988). However, characterization of vein densities is usually secondary to other mapping or sampling objectives; outcrops are not necessarily well exposed in all directions, a drill core being an extreme example, and generating two-dimensional vein maps is time intensive. For these reasons, vein data are frequently collected by using suitably oriented linear transects. The data in this investigation were similarly acquired. Intersections of veins with a transect define points on a line and are fractal if $0 < D < 1$.

The fractal dimension is obtained for linear vein transects by mapping the positions at which veins intersect transect lines. The map is then broken into equally spaced divisions of length $r$ and the number of divisions $N_r$ that contain one or more veins is recorded. After repetition for a range in $r$, $D$ is related to $r$ and $N_r$ by

$$D = \frac{d(\log N_r)}{d(\log r)}.$$

If $D$ is a constant between 0 and 1 for a range in $r$, the veins compose a homogeneous fractal set and are self-similar or scale invariant over the included length scales. The magnitude of $D$ is a measure of the manner in which the transect is filled by points; $D$ increases with increasing degree of space filling and decreasing clustering of the veins.

Fractal analyses were conducted on veins from metabasalt and marble outcrops and from hydrothermally altered diabases of Ocean Drilling Program Hole 504B. In the outcrop vein surveys, a tape measure was laid out normal to the vein strike. The locations of intersections of each vein were then recorded, along with their widths and the minerals filling them. The characterization of vein densities in drill core is analogous in that the axis of measurement is a line, but here it is the bore hole. For each set, $D$ was determined by linear least-squares analysis. Correlation coefficients $R$ reflect the set’s linearity, or self-similarity, not the uncertainty in $D$.

The range in $r$ that can be investigated is an intrinsic property of the data sets and is defined by upper and lower cutoffs (Chilès,
1988). The upper cutoff is the value of $r$ for which each segment is intersected by at least one vein. The lower cutoff is defined by the widths of the veins or the precision of the measurements of vein locations. Transect orientation can bias the inferred spatial structure of clustering (Terzaghi, 1965; La Pointe and Hudson, 1985; Veldke et al., 1990). To avoid this bias, the outcrops were evaluated carefully to ensure that the veins defined regularly oriented sets for which linear transects would be appropriate and that no veins were subparallel to the tape measure. For veins sampled by drill core, no such guarantees exist. The orientation of the hole is usually unknown, and the extent to which the axis of measurement may parallel vein attitudes is undetermined. However, the sampling biases inherent in vein and fracture orientations in vertical bore holes (Newmark et al., 1985) can be ignored if vein orientations are random, which is the case for Hole 504B (Dick et al., 1992).

**Wollastonite-Quartz Veins in Scalon Gulch Marbles**

Upper Paleozoic calc-silicates and marbles in the Scalon Gulch area of the Old Woman Mountains, southeastern California, were thermally metamorphosed in a screen $\sim 1$ km thick between $\sim 73$ Ma granite intrusions (Foster et al., 1989, 1992; Rothstein, 1990). Both lithologies contain abundant wollastonite, implying moderate to large time-integrated fluid fluxes and silica metasomatism during metamorphism. Subvertical veins trending 320-350 cut across the calc-silicates and marbles. The veins postdate the peak in metamorphism and are filled by wollastonite, quartz, or both. An 8.5 m survey line oriented 065 in wollastonite + calcite marbles of the Supai Formation intersected 89 veins yielding an average vein density of 10.5/m. The wollastonite-quartz veins are $\leq 1$ to 8 mm thick.

Figure 1A shows that $\log N_r$ is linear in $\log 1/r$ with $D = 0.46$ and $R = 0.994$. This means that the veins are self-similar over two orders of magnitude of $r$ between 0.01 and 1 m. The lower cutoff of 1 cm was defined by the maximum vein widths of 8 mm. Transects of greater length are required to determine if self-similarity continues to higher $r$.

**Actinolite-Chlorite Veins in Hole 504B**

Sheeted dikes from $\sim 1600$ to 2000 m below the sea floor were recovered from Hole 504B during Leg 140 of the Ocean Drilling Program. Dikes variably altered to green schist facies mineral assemblages are crosscut by randomly oriented hydrothermal veins (Dick et al., 1992). As noted by Dick et al. (1992), $\geq 98\%$ of the veins in the core contain actinolite, chlorite, or both, and $\leq 5\%$ albite, epidote, quartz, prehnite, titanite, and sulfides. These veins, referred to as actinolite-chlorite veins, are crosscut by rare epidote-quartz veins with trace sulfides. Because they formed later, the epidote-quartz veins were omitted from the analysis. The average density of actinolite-chlorite veins is 11.5/m.

During description of the veins in the drill core, the number of veins in each piece of core was noted rather than the exact locations of the veins relative to a reference line (Dick et al., 1992). This limits the minimum value of $r$ to the maximum length of recovered pieces, which was $\sim 10$ cm. Figure 1B shows that $\log N_r$ varies linearly with $\log 1/r$ for veins in the 56.9 m of diabases recovered from the $\sim 40$ cm interval drilled during Leg 140, giving $D = 0.81$ with $R = 0.995$. This linear variation suggests self-similarity over one order of magnitude of $r$.

Core samples reflect individual pieces from unknown but sequential positions within drilled intervals of known length. Errors are introduced into the fractal dimension of vein distribution because measurements include $\sim 1$ cm spacers added between pieces during curation. The added space makes the apparent $D$ larger than the actual $D$, but this effect is small for such large data sets (La Pointe, 1988) and does not influence the precision of the self-similarity.

**Epidote-Quartz Veins in Skaergaard Metabasalts**

Tertiary basalts in central East Greenland were contact metamorphosed by the Skaergaard intrusion (Bird et al., 1986; Manning and Bird, 1991; Manning et al., 1993). Veins filled by epidote and quartz are common in the aurole. Two traverses were conducted 1.5 and 2.0 km east of the intrusion where contact metamorphism led to greenschist facies assemblages. In both localities, the veins are $0.1-2$ mm wide and vertical, and have a narrow range of strike (070-090°). Figure 1C shows results for 57 veins intersected along a 30 m line striking 000 (1.9 veins/m). The strong linear correlation of $\log N_r$ in $\log 1/r$ over one order of magnitude of $r$ (Fig. 1C) shows that the vein distribution is statistically self-similar with a fractal dimension of 0.45 ($R = 0.980$). A second traverse 5.4 m long and oriented 355 intersected 55 epidote-quartz veins for an average vein density of 10.1/m. Figure 1D

![Figure 1](image_url)
shows that the veins are fractal \((0 < D < 1)\),
but there are two similarity dimensions: at
low \(r\), \(D = 0.25\) \((R = 0.981)\); at high \(r\), \(D = 0.63\)
\((R = 0.991)\). The two values of \(D\) mean
that the vein distributions scale with differ-
et fractal dimensions over different ranges
in \(r\). This could be a consequence of the two
inferred fracturing episodes with the same
mineral filling assemblage and orientation in
this part of the Skaergaard’s contact aureole
(Manning and Bird, 1991).

**FRAC TAL CLUSTERING OF
METAMORPHIC VEINS**

The mapped vein arrays in Figure 1, A–C,
are self-similar fractals over 1.5 to 2.5 orders
of magnitude in \(r\). Their self-similarity
means that the spatial structure is scale
invariant between the upper and lower cutoffs.
The veins in Figure 1D display two distinct
similarity dimensions. These fluid-flow
channels thus have two levels of spatial
structure, each of which is scale invariant
within a distinct range in \(r\).

For a given transect length, and number of
veins, the fractal dimension increases with
decreasing degree of clustering (La Pointe,
1988). Of the vein sets investigated, the out-
crop surveys have lower \(D\). These vein sets
display more clustering and larger unvined
regions. By contrast, the actinolite-chlorite
veins in Hole 504B yield the largest ob-
served \(D\) (0.81). The high \(D\) indicates a more
pervasive set with less space between veins.

The fractal clustering of metamorphic
veins can be appreciated by comparing the
observed distribution of veins at Scanlon
Gulch to the same number of purely random
veins and evenly spaced veins (Fig. 2). Ran-
dom veins differ from evenly spaced veins in
that random, uneven spacings result in
minor clusters. However, the natural pattern
is substantially more clustered than the purely
random pattern, presumably because initial
zones of weakness persist throughout frac-
turing events. The variation in log \(N_r\) with
log \(1/r\) for evenly spaced veins displays two
linear segments (Fig. 3). Where log \(1/r\) is
between \(-1\) and 0, log \(N_r\) is constant and
\(D = 0\), the topologic dimension of
the veins; where log \(1/r\) is between \(-2\) and \(-1\),
log \(N_r\) decreases linearly such that \(D = 1\),
the topologic dimension of the transect.
Evenly spaced veins therefore do not define a
fractal set. Also shown in Figure 3 is the
variation in \(N_r\) with \(1/r\) for 89 veins distrib-
uted randomly over 8.5 m. Near the upper
and lower cutoffs, \(D\) approaches limits of
1 and 0, respectively. Between these limits, \(0 < D < 1\)
and \(D\) varies continuously with \(r\).
Thus, random distributions are also not self-
similar. A random distribution would imply
that fractures were completely independent
of one another. The difference between the
synthetic random clustering and the self-
similar clustering of the observed distribu-
tion emphasizes that natural fluid-flow chan-
nels are interdependent.

**IMPLICATIONS FOR METAMORPHISM**

The dominant sites of metamorphic fluid
flow are fractures at some scale (e.g.,
Brenan, 1991). At microscopic scales, these
fractures are represented by grain bound-
aries and intragranular cracks commonly re-
corded as healed fluid-inclusion arrays. At
larger scales, flow channels are represented
by macroscopic veins. Independent of the
scale of observation, metamorphic veins
record the time-integrated history of flow for
all or part of a metamorphic event. That ob-
served macroscopic vein sets are fractal pro-
vides a framework for understanding the
spatial and temporal patterns of metamor-
phic fluid flow in the study areas.

Conceptual models for the evolution of
fluid flow during metamorphic events sug-
gest that fluid is pervasively distributed in
microscopic flow channels in the rock ma-
trix during prograde metamorphism. This
permits the extensive mineral-fluid reaction
that characterizes this part of the event. By
contrast, localized mineral-fluid reaction
during later retrograde metamorphism sug-
gests that fluids become channeled into
larger macroscopic fractures as the rocks
cool. Treating metamorphic flow channels
as fractal clusters is a better framework than
the end-member view of channeled vs.
pervasive flow in that it provides a quantitative
measure for comparing and contrasting fluid
flow in different areas and at different scales.
Comparing \(D\) for crosscutting veins allows
quantification of the transition from perva-
sive flow (high \(D\)) to more channeled flow
(low \(D\)).

The recognition of fractal vein clustering
also aids in the interpretation of the spatial
patterns of the progress of metamorphic re-
actions. For example, if a water-rich fluid
saturated with quartz were flowing in frac-
tured calcite marbles at amphibolite facies
conditions, the chemical potential gradient
in the thermodynamic component SiO\(_2\)
would lead to diffusive haloes of wollasto-
nite about each vein. This is a useful simpli-
fied model for fracture-controlled metamor-
phism at Scanlon Gulch. Assume in this
illustration that fluxes were sufficient to
cause 5 cm wollastonite haloes around each
fracture (Fig. 2, stippled pattern) via the re-
action CaCO\(_3\) + SiO\(_2\)(aq) −→ CaSiO\(_3\) + CO\(_2\).
A simple measure of the extent of transfor-
mation of the outcrop to wollastonite is the
reaction progress variable, \(\xi\), which can be
calculated from the number of moles of
wollastonite divided by its stoichiometric reac-
tion coefficient in the model reaction (e.g.,
Ferry, 1986). Assuming negligible Ca solu-
bility, there will be a mole-for-mole trans-
formation of calcite to wollastonite in the hy-
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