# Elastic Thickness Estimates for Venus from Line of Sight Accelerations

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A new method of obtaining the transfer function, or admittance,  $Z^{f}$ , between the Fourier transforms of topography and gravity of a planet is proposed that uses the line of sight (LOS) Doppler velocities directly. The expected LOS accelerations are first calculated from the spherical harmonic coefficients of the topography, the latitude, longitude, and height of the spacecraft, and the direction of Earth viewed from the planet. The admittance can then be obtained using standard signal processing methods, by comparing the LOS acceleration calculated from the topography with the time derivative of the observed LOS Doppler velocities. This method is applied to the Magellan data from cycle 4 for Atla and shows that the short wavelength behavior of the admittance is that expected from an elastic layer whose thickness  $T_e$  is 30 ± 5 km. The main contribution to the short wavelength gravity field comes from the large volcanoes Ozza, Maat, and Sapas Montes. Comparison with admittance estimates from spherical harmonic gravity fields and from local inversions shows that these methods produce estimates of  $Z^{f}(k)$  that do not fit those expected from a simple flexural model. The  $T_e$  values from Beta and Ulfrun of 27.5 and 33 km are similar to that of Atla, whereas Dali (12 km) gives a smaller value. No reliable value can yet be estimated from Aphrodite, probably because the topography is poorly determined. These estimates of  $T_{\rm e}$  cover the same range as those from flexural modeling of topography associated with coronae. © 1997 Academic Press

# 1. INTRODUCTION

On Earth the deformation of the lithosphere is strongly controlled by the thickness of that part which can sustain elastic stresses for long periods. The thickness of this elastic layer varies, from less than about 5 km on spreading ridges to 30 km beneath old ocean basins and 20 km beneath shields. Its thickness is generally estimated by using the correlation between gravity and topography, either using spectral methods (Lewis and Dorman 1970, McKenzie and Bowin 1976, Forsyth 1985) or by direct modeling in the space domain (Gunn 1943, Walcott 1970, Watts and Cochran 1974, Watts 1978). It is of great importance to estimate the thickness of the elastic layer on Venus. It is, however, only straightforward to do so if its thickness is sufficiently large to affect the gravity field at those wavelengths that are well determined from Magellan's orbit. The surface gravity field of wavelength  $\lambda$  is reduced by a factor of  $\exp(-2\pi h/\lambda)$  at the orbital height h, or approximately  $\exp(-6h/\lambda)$ . Furthermore, the spectrum of the gravity field is red and is therefore dominated by the long wavelength components. These two effects combine to prevent reliable estimates of the surface gravity field of Venus at wavelengths shorter than about 400-500 km. On Earth the elastic effects only begin to dominate over the convective gravity anomalies at these and shorter wavelengths. Sandwell and Schubert (1992) estimated the elastic thickness on Venus from the topography of coronae to be 15–40 km. and Johnson and Sandwell (1994) obtained values of 12-34 km. These values are similar to those from stable regions on Earth, and are therefore likely to be difficult to estimate reliably using the surface gravity field. Attempts to do so (Phillips 1994 and written communication 1996, Smrekar 1994) using the spherical harmonic description of the gravity field, or using inverse methods, have produced a considerable range of values, probably because of problems with noise and with leakage of longer wavelength energy to shorter wavelengths.

For instance Smrekar (1994) estimated the behavior of  $Z^{f}(k)$  using singular value decomposition. She shows estimates for a number of wavenumber bands corresponding to wavelengths longer than 400 km and uses these to estimate a value of  $T_{e}$  of  $30 \pm 5$  km for Atla. However her values of  $Z^{f}$  increase by more than a factor of 2 between the band 500–600 km and 400–500 km (see below). This feature is not found in any terrestrial examples (McKenzie and Fairhead 1997), nor in any of the estimates made below. These problems have led to disagreements about the thickness of the elastic part of the lithosphere on Venus that are unlikely to be easily resolved by using the surface gravity field.



FIG. 1. Definitions of  $\psi$ , the Earth–Sun–Venus angle, and  $\Theta$ , the angle between the normal to Magellan's orbit and the Earth–Venus vector.

The principal purpose of this paper is to apply some simple ideas from potential theory and signal processing to determine the relationship between the gravity and topography at short wavelengths where the signal to noise ratio is poor, and hence to estimate the elastic thickness beneath various regions of Venus. The same approach can be used to study the accuracy of the spherical harmonic models of the gravity field. The principle of the method

we use is a straightforward extension of that which McKenzie and Bowin (1976) proposed for marine gravity and topography. The topography of Venus is in general well determined (but see Fig. 17 below), and the spherical harmonic coefficients for the planet have been calculated by Rappaport and Plaut (1994). It is therefore easy to obtain the coefficients of the gravity field for any given transfer function, often called the admittance, between gravity and topography. These coefficients can then be used to calculate the three components of the expected gravity field at the position of Magellan, and hence the expected line of sight (LOS) acceleration. Standard methods of signal processing can be used to extract that part of the observed LOS acceleration which is coherent with the acceleration calculated from the topography. Since the problem is linear, we calculated the LOS acceleration using a real admittance of 1 mGal/km that was independent of frequency and then estimated the true admittance by Fourier transforming the observed and calculated accelerations. An important advantage is that the method uses upward continuation, which is always stable. No artifacts therefore arise due to the methods used to stabilize the unstable inversion. The necessary potential theory is described in Section 2, and the data processing in Section 3. Sources of noise are discussed in Section 4. Section 5 compares the results



FIG. 2. LOS accelerations, low pass filtered with a Kaiser filter with a cutoff of 10 sec, for two face-on and two edge-on orbits. Number at top left is orbit number; numbers along x-axis are latitude and longitude. The thin line is the observed, and the dotted line is the calculated LOS acceleration from the spherical harmonic topography (l = m = 360) and Eq. (12), assuming a constant admittance of 50 mGal/km.



FIG. 3. As Fig. 2, except for two inferior conjunction ( $\psi \simeq 0^{\circ}$ ) face-on orbits, compared with two edge-on superior conjunction ( $\psi \simeq 180^{\circ}$ ) passes.

obtained here with those from Smrekar (1994) and Phillips (1994), and examines the reliability of estimates of the admittance obtained from spherical harmonic models of the gravity field. Section 6 then uses the observed LOS acceleration to estimate the elastic thickness of various regions of Venus.

#### 2. POTENTIAL THEORY

An important geophysical observable is the transfer function  $Z^{f}(k)$  between the Fourier transforms of the gravity field  $\overline{g}(k)$  and the topography  $\overline{h}(k)$ ,

$$\overline{g}(k) = Z^{f}(k)\overline{h}(k), \qquad (1)$$

where  $k = 2\pi/\lambda$  is the wavenumber.  $Z^f$  is usually assumed to be isotropic, and on Earth is known to be a function of position. The estimates of  $Z^f$  that have been reported for Venus have all used the surface gravity field, represented either by spherical harmonic expansions or by point masses, obtained by inversion of the LOS accelerations (McKenzie 1994, Phillips 1994, Grimm 1994, Smrekar 1994). These methods are limited to those wavelengths that are sufficiently long for the signal from the surface gravity at the height of the spacecraft to dominate the noise resulting from instrument effects and path length variations caused by the ionosphere and by the solar wind. They must be stabilized by some type of regularization to suppress short wavelength instabilities that arise through downward continuation.

An alternative approach to the problem of estimating  $Z^{f}(k)$  is to calculate the expected LOS acceleration of Magellan itself from the surface topography. If  $C_{lm}^{tr}, S_{lm}^{tr}$  are the normalized harmonic coefficients of the topography, then surface topography  $t(\theta, \phi)$  is given by

$$t = a \sum_{l=2}^{\infty} \sum_{m=0}^{l} \overline{P}_{lm}(\theta) [C_{lm}^{\prime \prime} \cos(m\phi) + S_{lm}^{\prime \prime} \sin(m\phi)], \quad (2)$$

where  $\overline{P}_{lm}(\theta)$  are the normalized Legendre and associated Legendre polynomials. The gravitational potential  $U(r, \theta, \phi)$  is given by

$$U = \sum_{l=2}^{\infty} \sum_{m=0}^{l} \left(\frac{a}{r}\right)^{l+1} \overline{P}_{lm}(\theta) U_{lm}(\phi),$$
(3)

where

$$U_{lm}(\phi) = C_{lm}^G \cos(m\phi) + S_{lm}^G \sin(m\phi).$$



**FIG. 4.** (a) Power spectra for LOS accelerations for six groups of orbits at intervals of  $\psi$  of 30°. (b) The continuous solid line shows the power calculated from Eq. (17). The crosses show the power between periods of 36 and 88 sec from the spectra in (a), with the length of the vertical line showing the standard deviation, plotted so that the value for  $\psi = 150^{\circ}$  lies on the theoretical curve. The position of the dotted line marked "instrumental noise" is estimated from the observations.

*a* is the radius of the planet and  $C_{lm}^G$ ,  $S_{lm}^G$  are constants. The vertical component of the gravitational acceleration at the surface is

$$-\left[\left(\frac{\partial U}{\partial r}\right)\right]_{r=a}$$

$$= \frac{1}{a} \sum_{l=2}^{\infty} \sum_{m=0}^{l} (l+1)\overline{P}_{lm}(\theta) [C_{lm}^{G} \cos(m\phi) + S_{lm}^{G} \sin(m\phi)].$$
(4)

If the admittance is isotropic, it can depend only on the degree l of the spherical harmonic and must be independent of m. Combining Eqs. (1), (2), and (4) gives

$$(C_{lm}^G, S_{lm}^G) = \frac{Za^2}{l+1} (C_{lm}^{\prime t}, S_{lm}^{\prime t}).$$
 (5)

The vector acceleration  $\mathbf{g}$  is given by

(

$$\mathbf{g} = \nabla U = \frac{\partial U}{\partial r} \,\hat{\mathbf{r}} + \frac{1}{r} \frac{\partial U}{\partial \theta} \,\hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial U}{\partial \phi} \,\hat{\phi},$$

where  $\hat{\mathbf{r}}$ ,  $\hat{\theta}$ , and  $\hat{\phi}$  are unit vectors pointing upward, southward, and eastward, respectively. At the spacecraft height h,

$$\begin{bmatrix} \left(\frac{\partial U}{\partial r}\right) \end{bmatrix}_{r=a+h} = -\frac{1}{a} \sum_{l=2}^{\infty} \sum_{m=0}^{l} (l+1) \overline{P}_{lm}(\theta) A_l U_{lm}(\phi) \quad (6)$$
$$\begin{bmatrix} \frac{1}{r} \left(\frac{\partial U}{\partial \theta}\right) \end{bmatrix}_{r=a+h} = \frac{1}{a} \sum_{l=2}^{\infty} \sum_{m=0}^{l} \frac{d\overline{P}_{lm}(\theta)}{d\theta} A_l U_{lm}(\phi) \quad (7)$$
$$1 \quad \left(\frac{\partial U}{\partial \theta}\right) \end{bmatrix}$$

$$\lfloor r\sin\theta \setminus \partial\phi \rfloor_{r=a+h}^{\infty} = \frac{1}{a} \sum_{l=2}^{\infty} \sum_{m=0}^{l} m \overline{P}_{lm}(\theta) A_l [-C_{lm}^G \sin(m\phi) + S_{lm}^G \cos(m\phi)],$$

$$(8)$$

where

$$A_l = \left(\frac{a}{a+h}\right)^{l+2}$$

and  $d\overline{P}_{lm}(\theta)$  is calculated from the recurrence relationship

$$\frac{d\overline{P}_{lm}(\theta)}{d\theta} = \frac{l}{\tan\theta} \overline{P}_{lm}(\theta) - \left[\frac{(2l+1)(l^2-m^2)}{2l-1}\right]^{1/2} \overline{P}_{l-1,m}(\theta).$$
(9)

The LOS acceleration is then obtained by projecting  $\mathbf{g}$  onto the line of sight vector  $\mathbf{d}$ , where

$$\mathbf{d} = (\cos \phi_{\rm e} \cos \lambda_{\rm e}, \sin \phi_{\rm e} \cos \lambda_{\rm e}, \sin \lambda_{\rm e}), \qquad (10)$$

where  $\lambda_e$ ,  $\phi_e$  are the latitude and longitude of the Earthpointing vector in Venus planetocentric coordinates. In the same coordinates

$$\hat{\mathbf{r}} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$$
$$\hat{\theta} = (\cos\phi\cos\theta, \sin\phi\cos\theta, -\sin\theta)$$
(11)
$$\hat{\phi} = (-\sin\phi, \cos\phi, 0).$$



FIG. 5. Contour map of the topography of Atla Regio, using the Magellan altimetry data (Ford and Pettengill 1992) and a sinusoidal equal area projection.

The calculated LOS acceleration  $g_c$  is then given by

$$g_{\rm c} = \left[\frac{\partial U}{\partial r}\left(\mathbf{\hat{r}}\cdot\mathbf{d}\right) + \frac{1}{r}\frac{\partial U}{\partial\theta}\left(\hat{\theta}\cdot\mathbf{d}\right) + \frac{1}{r\sin\theta}\frac{\partial U}{\partial\phi}\left(\hat{\phi}\cdot\mathbf{d}\right)\right]_{r=a+h}.$$
(12)

Two special cases arise when the orbit is viewed face-on, with  $\hat{\mathbf{r}} \cdot \mathbf{d} = 0$ , and when it is polar. In the first case  $\hat{\theta} \cdot \mathbf{d}$ and  $\hat{\phi} \cdot \mathbf{d}$  are in general non-zero, and hence  $g_c = 0$  for the whole orbit if and only if all  $C_{lm}^G, S_{lm}^G$  are zero. If the orbit is both face-on and polar,  $\hat{\mathbf{r}} \cdot \mathbf{d} = \hat{\theta} \cdot \mathbf{d} = 0$  and the zonal harmonic coefficients with m = 0 make no contribution to the observed or the calculated LOS acceleration. However, the third term in Eq. (12) is non-zero.

# 3. DATA PROCESSING

Good coverage of LOS velocity was obtained during cycles 4, 5, and 6. During cycle 4 Magellan's orbit was strongly elliptical, whereas that for cycles 5 and 6 was approximately circular, with an apoapse of  $\sim$ 600 km (cycle 5) and  $\sim$ 400 km (cycle 6) and a periapse of  $\sim$ 180 km. The

spacecraft position is known to an accuracy of about 100 m, and therefore location errors have an insignificant effect on the calculated LOS acceleration. The acceleration is calculated at the spacecraft location.

The first attempt to apply the methods described above to obtain  $T_e$  from LOS accelerations used data from cycles 5 and 6, because cycle 4 produced poor estimates of  $Z^f(k)$ . However Konopliv and Sjogren (written communication, 1997) have now used the complete Magellan data set to determine the gravity field to l = m = 120. The resulting improvements in orbit have eliminated the earlier problems, and only data from cycle 4 is discussed below. A similar analysis is in progress using cycles 5 and 6.

The first step is to use the spherical harmonic coefficients of topography to calculate the three components of the gravitational acceleration at the position of the spacecraft. Rappaport and Plaut (1994) used a gridded data set to obtain values of the coefficients by integration. When their values were used to obtain the topography for Atla, a small phase difference, corresponding to an offset of about 5 km, was found between topography obtained by direct gridding of the altimetry and that from the spherical harmonic coefficients. Partly to eliminate this offset, and partly



**FIG. 6.** (a) Admittance  $Z^f$  of the Atla area (see Fig. 5), obtained from the observed and calculated two-dimensional gridded LOS acceleration (see Figs. 8 and 9). Details of windowing are given in McKenzie and Fairhead (1997). The vertical bars show one standard deviation, and the solid curve shows the best fitting flexural model using Eq. (18) of McKenzie (1994) with  $T_e = 28$  km, crustal density 2.67 Mg m<sup>-3</sup>, and thickness of 16 km. (b) The misfit  $H^f(T_e)$  is defined by Eq. (18) for the data in (a) for wavelengths between 150 and 500 km. (c) The coherence between the observed and calculated LOS acceleration. (d) and (e) show  $Z^f$  for two terrestrial examples discussed by McKenzie and Fairhead (1997), recalculated using wavelengths between 250 and 500 km. The value of  $T_e$  for the whole of Siberia is slightly large than those of 19 and 15.5 km obtained by McKenzie and Fairhead (1997) for two smaller regions.

to include all available altimetry, a new set of topographic coefficients were determined, to l = m = 360. All coefficients were used to calculate the acceleration at every point.

Most of the orbits calculated from the l = m = 120

gravity field give the position, velocity, and Doppler residual every 2 sec. The orbital velocity is about 8.1 km sec<sup>-1</sup> close to periapse during cycle 4, and therefore the aliasing wavelength is about 32 km. Since the spacecraft elevation was never less than 180 km, it is essentially unaffected by





FIG. 7. As for Fig. 6, but for the individual orbits used to obtain the gridded data in Figs. 6, 8, and 9. The best fitting flexural model has a  $T_e$  of 33 km.

gravity anomalies with wavelengths shorter than about 100 km. It is therefore useful to low pass filter the data and to resample it using a longer interval, in order to reduce the number of points at which the gravity field must be calculated. The exact form of the filter used is not important provided it has low pass band ripple and that the signal is strongly attenuated at the new aliasing frequency and higher frequencies. We used a Kaiser filter with 25 coefficients, a cutoff at 10 sec, and resampled every 4 sec. The line of sight acceleration profile data records (LOSAPDRs) for cycle 4 allow the calculation of the line of sight velocity  $v_m$  of the spacecraft, given the acceleration due to the nominal gravity field (MGN120PSAAP), and also give the two way Doppler residual  $\Delta f$ . The Doppler

residual was first converted to a LOS velocity  $\Delta v$  using

$$\Delta v = c \Delta f / 2f_0, \tag{13}$$

where *c* is the velocity of light and  $f_0$ , = 8.43 GHz, is the reference frequency.  $\Delta v$  was then combined with  $v_m$  to obtain a total LOS velocity  $v = v_m + \Delta v$ . The LOSAPDRs also list LOS accelerations calculated by differentiating cubic spline fits to the LOS velocities. These should not be used for modeling, since spline fitting removes the short wavelength information that is required.

FIG. 8. Contours of the observed LOS acceleration,  $g_0$ , for Atla Regio in Fig. 5. The green lines mark the rift boundaries and the blue lines the volcanic edifices, taken from Senske *et al.* (1992).

FIG. 9. Contours of the calculated LOS acceleration,  $g_c$ , calculated from Eq. (12) using an admittance of 1 mGal/km and l = m = 360 spherical harmonic topography, for Atla Regio.





**FIG. 11.** Profiles of  $g_o - 50g_c$  along the two lines shown in Fig. 10, to show the shape of the elastically supported loads.

The LOS velocity v and acceleration g are related by

$$\frac{dv}{dt} = g. \tag{14}$$

In the frequency domain, Eq. (14) gives

$$2\pi i F \overline{v} = \overline{g},\tag{15}$$

where F, the frequency in the time domain, is related to f, that in the space domain, through f = F/V where V is the orbital velocity. Hence the admittance between gravity and topography can be obtained directly from the Fourier transforms of the observed velocities and the calculated accelerations. As Eq. (15) shows, the admittance between  $\overline{g}$  and  $\overline{v}$  should be imaginary. Equation (15) gives satisfactory estimates of  $Z^{f}(k)$  where the signal to noise ratio is large ( $\geq 1$ ) and when long tracks are used. However, it does not yield reliable local estimates using short tracks because of spectral leakage. The spectrum of the surface gravity is already red, and upward attenuation makes it even more so at the orbital height. As Eq. (15) shows, the Fourier transform of the velocity and that of the gravity are related by 1/F, so the velocity spectrum is even redder than that of the gravity. It is therefore desirable to prewhiten the observations before Fourier transformation. The obvious method of doing so is to use the LOS velocities  $v_1 \dots v_n \dots v_N$ , measured at intervals  $\Delta t$ , to calculate the LOS accelerations  $g_1 \dots g_n \dots g_N$  using

$$g_n = \frac{v_{n+1} - v_{n-1}}{2\Delta t}$$
(16)

to obtain a properly centered value of  $g_n$ . Values of  $g_n$  were only used when the spacecraft height was less than 400 km.

All spectra were calculated using the multitaper method (Thomson 1982, Johnson 1994, McKenzie and Fairhead 1997) with three windows for one-dimensional transforms and three in each direction, or nine altogether, for twodimensional transforms. All estimates of the admittance used the accelerations calculated from the topography as input, and therefore assume that the topographic signal is free from noise.

For most spacecraft in orbit around most planets the admittance must be obtained by comparing one-dimensional Fourier transforms of calculated and observed accelerations of each orbit. Such an approach has the disadvantage that variations at right angles to the spacecraft velocity vector are not properly taken into account. However, because of the slow rotation of Venus and the orbit chosen for Magellan, each successive path of the spacecraft is adjacent to the previous one on the planet and forms a series of curves progressing from west to east, by about  $0.2^{\circ}$ /cycle during cycle 4, covering the whole planet in about one venusian year. This behavior can be used to construct two-dimensional grids of LOS observed and calculated accelerations, which can then be windowed and transformed in two dimensions. The contour maps of the resulting grids are useful, though they are not easily interpreted directly. Because the line of sight to the Earth is continuously changing, such maps combine contributions of all three components of the acceleration, and the contribution each component makes changes both along each orbit and from east to west. Since both the observed and calculated accelerations are affected in the same way, they can be directly compared. But such maps do not show the vertical component of the acceleration, and the anomalies they show are generally displaced with respect to surface features.

## 4. GEOMETRIC EFFECTS

The observed LOS acceleration is affected by the viewing geometry and by the interplanetary medium through

**FIG. 10.** Contours of  $g_o - 50g_c$ , to show the locations of the elastically supported anomalies. The arrowed lines show the location of the profiles in Fig. 11.

FIG. 12. The residual topography, h - g/50, where g is the gravity field in mGal calculated from mgnp60fsaap to l = m = 60 and h is the gridded Magellan altimetry in km. The three red arrowed lines show the locations of the profiles plotted in Fig. 13.



**FIG. 13.** Three profiles of residual topography used to estimate the elastic thickness from footwall uplift. Bold curve is the data; dashed curve is the flexural profile assuming a  $T_e$  of 16 km. The rms misfit  $H^s(T_e)$  (Eq. (18)) is shown in (b).

which the X band signal travels. The viewing geometry is conveniently described in terms of an angle,  $\Theta$ , between the normal to the orbital plane of the spacecraft and the vector pointing toward the Earth (Fig. 1). Orbits with  $\Theta$ close to  $0^{\circ}$  are described as face-on, ones with  $\Theta$  about  $90^{\circ}$ as edge-on. Face-on orbits are often described as having poor geometry for gravity field determination. This statement is in general not correct. Though the LOS accelerations measured by face-on orbits are not affected by the radial component of gravity, they show large variations resulting from the tangential gravity field. Figure 2 shows four profiles, two approximately face-on and two edge-on, of calculated and observed accelerations plotted at the same horizontal scales. The face-on profiles are longer, because the entire orbit is visible from Earth when  $\Theta = 0^{\circ}$ . They therefore contain more, rather than less, information about the gravity field than do the edge-on orbits. There is in general good agreement between LOS accelerations calculated from the topography for a value of  $Z^{f}$  of 50 mGal/km and the observed face on profiles, except where the topography is compensated.

The other geometric effect that is considerably more troublesome is the effect of the ionized plasma that originates from the Sun. Close to superior conjunction the angle  $\psi$  in Fig. 1 is close to 180°, and the path is subject to large variations in electron density. Figures 3b and 3d show that

the Doppler signals then become very noisy. Corresponding plots of the power spectra of the apparent LOS acceleration in Fig. 4a show a strong dependence on  $\psi$  at short wavelengths close to superior conjunction, when  $\psi = 180^{\circ}$ , but little dependence when  $\psi \leq 120^{\circ}$ . This behavior is caused by two different effects. Where the signal from the spacecraft passes close to the Sun, path-length variations produced by fluctuations in electron density dominate the Doppler variations. At smaller values of  $\psi$  instrumental noise in the up- and down-link system is more important. A crude model of this behavior assumes that the plasma effects depend simply on the density of electrons integrated along the path, and that the electron density is proportional to  $1/r^2$  where r is the distance from the Sun. The resulting expression for the angular dependence of the integrated density  $I_P$  is

$$I_P = A\psi(\beta^2 - 2\beta\cos\psi + 1)/\sin\psi, \qquad (17)$$

where  $\beta = 0.72$  is the ratio of the radius of the orbit of Venus to that of Earth, and A is independent of  $\psi$ .  $I_P$  is plotted in Fig. 4b, with noise estimates from Fig. 4a, which have been scaled so that the value for  $\psi = 150^{\circ}$  lies on the curve. Variations of electron density dominate the behavior at large values of  $\psi$  and instrumental noise at smaller values (Asmar, written communication). In fact the Doppler signals do not depend on the integrated electron density, but on its fluctuations (Armstrong *et al.* 1979, Woo *et al.* 1995), and therefore the expression used in Fig. 4b is too simple. However, the agreement between the observed and calculated  $\psi$  dependence suggests that the sources of noise have been correctly identified. During cycle 4  $|\psi| <$ 120°, and therefore the noise was dominated by instumental effects.

## 5. ATLA

Both Phillips (1994) and Smrekar (1994) have used the gravity data from Atla to estimate  $T_e$ . To compare the results obtained here with theirs we used the same region as Phillips,  $-10^{\circ}$  to  $-25^{\circ}$ N,  $180^{\circ}$  to  $215^{\circ}$ E. We first discuss the results obtained by using the method outlined above, and then compare them with the earlier studies. Atla is a large elevated region (Fig. 5) crossed by a number of rifts, and on which three large volcanoes have been constructed. It is generally believed to be the surface expression of a large upwelling plume. The boundaries of the rifts and volcanic edifices shown in Fig. 5 are from Senske *et al.* (1992).

Figure 6 shows the two-dimensional admittance, the coherence between observed and calculated LOS acceleration. The misfit  $H^f$  of theoretical admittance curves  $Z_c^f(k)$  to the observed values of  $Z_c^f(k)$  with standard devia-



FIG. 14. Open circles are admittance estimates for Atla from the gridded topography in Fig. 5 and observed LOS acceleration (see Fig. 6). Solid dots and lines show estimates and uncertainty of the admittance, respectively, from the gridded topography and four different spherical harmonic gravity models. The dotted line is theoretical admittance curve for  $T_e = 45$  km, the solid bold line for  $T_e = 28$  km, calculated in the same way as for Fig. 6. (a) Using spherical harmonic model mgnp60fsaap, (b) mgnp75hsaap, (c) mgnp90lsaap, (d) mgnp120psaap, (e) from Smrekar (1994, Fig. 11c).

tion  $\Delta Z_o^f(k)$  (McKenzie and Fairhead 1997) in the wavelength range 150–500 km is

$$H^{f} = \left[\frac{1}{N}\sum \left[(Z_{o}^{f} - Z_{o}^{f})/\Delta Z_{o}^{f}\right]^{2}\right]^{1/2},$$
 (18)

where N is the number of wavenumber bands used. Equation (18) shows that estimates of  $Z_o^f$  at short wavelengths contribute little to the value of  $H^f$ , because  $\Delta Z_o^f$  is large. Furthermore both  $Z_o^f$  and  $Z_c^f$  are independent of  $T_e$  at short wavelengths, because the topography is uncompensated. Therefore short wavelength estimates of  $Z_o^f$  have



**FIG. 15.** Tracks from cycle 4 at elevations of 400 km or less, showing the boxes used to estimate the values of  $T_e$ , together with the values obtained in km. The approximate values of  $|\psi|$  for the tracks of cycle 4 are shown as a function of the longitude where the track crosses the equator, together with the longitude of the start and end of this cycle.

little effect on the estimated value of  $T_{\rm e}$ , even when they are well determined. If the observations are modeled correctly,  $H^f \simeq 1$ , compared with a value of 0.78 for  $T_e = 28$ km. At wavelengths longer than about 500 km convective effects become important, and  $Z^f \simeq 50$  mGal km<sup>-1</sup>. The observed behavior of  $Z_{o}^{f}$  as a function of wavelength, the quality of the fit, and the behavior at long wavelengths closely resembles the data from regions on Earth for which good data are available (McKenzie and Fairhead 1997).  $Z^{f}$  for two such regions, Hawaii and Siberia, are illustrated in Figs. 6d and 6e. The Siberian data are noisier than those from Atla, which they otherwise closely resemble, probably because of erosion. Figure 7 shows a plot similar to Fig. 6, but treats the data as a number of one-dimensional series. As expected the estimates of  $Z^{f}$  are somewhat noisier, because the number of spectral estimates is smaller. The value of  $T_e$  that best fits the data is also slightly larger, 33 instead of 28 km, probably because of the spectral leakage to longer wavelengths that results when two-dimensional data are sampled with one-dimensional profiles. However, the comparison between Figs. 6 and 7 suggests that the method will give reliable estimates of  $T_{\rm e}$  even when the data cannot be converted to two-dimensional grids.

The advantage of estimating  $Z^{f}$  from the observed and calculated LOS acceleration is well illustrated by the results from Atla. For instance, in the wavelength range 230–250 km the multitaper approach gives 1152 spectral estimates

of both the observed and the calculated two-dimensional LOS accelerations, and their coherence  $\gamma^2$  is 0.052 and the signal to noise ratio is 1 to 20. The source of noise in this wavenumber band is likely to be instrumental (Fig. 4). On Earth the coherence in the same wavenumber band in eastern Siberia is 0.63 (McKenzie and Fairhead 1997), and the true coherence on Venus is likely to be larger. On Earth the coherence is reduced by low density sediments filling topographic lows. Because of the absence of erosion, this effect is likely to be less important on Venus. Because of the low coherence, it is unlikely that reliable estimates of the gravity field in this wavenumber band can be obtained from the Magellan data. Nonetheless a reliable estimate of  $Z^{f}(k)$  of 99.6  $\pm$  17.3 mGal/km can be obtained by the method used here, because so many spectral estimates are available. This value agrees well with 106.4  $\pm$ 7.7 mGals/km from eastern Siberia. In both regions wavelengths of 230–250 km are sufficiently short, and  $T_{\rm e}$  is sufficiently large, that the topography is essentially uncompensated.

An important problem with spectral methods is that they provide little information about the location of the features responsible for the observed behavior. Yet such information is clearly of considerable geological importance. In general, spectral methods that can extract information when the coherence is small cannot also yield spatial information. In the case of Atla, however,  $Z^{f}$  increases to values larger than 50 mGal km<sup>-1</sup> when the coherence is about 0.8. Maps of the gridded observed LOS acceleration  $g_o$  (Fig. 8) and that calculated using 1 mGal/km,  $g_c$ , (Fig. 9) were therefore produced, together with a residual map of  $g_o - 50g_c$  (Fig. 10) to emphasize those features with values of  $Z^f$  that differed from the convective value of 50 mGal/km. Though the residual map is somewhat noisy, it clearly shows features associated with the three large volcanoes, and also a region extending from 10°N to 20°N, 200°E. Two profiles constructed from Fig. 10 are shown in Fig. 11 and suggest that volcanic loads are responsible for the gravity signals that allow  $T_e$  to be estimated.

It is of interest to discover whether the estimates of  $T_e$ from gravity agree with those from topography. Unlike the coronae studied by Sandwell and Schubert (1992) and by Johnson and Sandwell (1994), where the flexural loads were probably imposed by thrusting, the major tectonic features in the Atla Regio are normal faults. However, these also show flexural effects, in the form of footwall uplift produced by unloading as the normal faults move. Such features are more obvious if the convective topography is removed, by calculating h - g/50, to produce the residual topography in Fig. 12. For reasons discussed below, the gravity coefficients from Konopliv and Sjogren (1994), extending only to l = m = 60, were used to calculate g. Figure 12 shows that footwall uplift is associated with many of rift flanks. Three profiles used to estimate  $T_{\rm e}$  are shown in Fig. 13, and give a value of 16 km. Further studies and comparison with terrestrial examples are required to discover whether the rifted regions have smaller values of  $T_{\rm e}$  than do the off-rift volcanic areas. There is some evidence for such effects from East Africa (Ebinger et al. 1989).

These results can now be compared with those from spherical harmonic models, and with local inversions that use various methods to stabilize downward continuation. Figure 14 shows estimates of  $Z_{0}^{f}$  from four spherical harmonic models, together with a curve calculated using the value of  $T_{\rm e}$  of 45 km estimated by Phillips (1994) and the curve and estimates from the method proposed here. The low values of  $Z_{o}^{f}$  at long wavelengths result from the removal of the means from both the gravity and topography. The major difference between the estimates of  $Z_{0}^{f}$  from spherical harmonics and those obtained here is for wavelengths shorter than 500 km, where that from the harmonics decreases and that from the direct method increases. This behavior is likely to result from the methods used to stabilize the inversion. It is, however, surprising that the inversion methods are not able to retrieve the gravity field at wavelengths between 300 and 500 km, where the coherence is greater than 0.5. Though Phillips' estimate for  $T_e$  of 45 km fits the spherical harmonic estimates at wavelengths longer than about 700 km, the calculated curve lies outside the standard deviation of all estimates in Fig. 6 and is therefore too large. Phillips (written communication, 1997) now believes that  $T_e = 25$  km agrees better with the observations than does his earlier estimate. Smrekar's (1994) estimates of  $Z_o^f$  obtained by local inversion are plotted in Fig. 14e with those obtained here for comparison. She increased the estimates of  $Z_o^f(k)$  she obtained from the spectra of her local inversion by 10 mGal/km to make them agree with the estimates from spherical harmonics. The cause of this offset is unknown (Smrekar 1994, caption to her Fig. 10), and it is unclear whether it is also present at the short wavelengths she uses to estimate  $Z^f$ . Though her estimates of  $Z_o^f(k)$  is quite different, the value of  $T_e$  of  $30 \pm 5$  km she obtained from the two values of  $Z_o^f$  for wavelengths between 400 and 500 km is the same.

Phillips (written communication, 1997) has questioned whether the topography used to estimate  $Z^{f}$  at wavelengths less than 500 km is supported elastically, as we have assumed here, rather than by convection. He has expressed particular concern that the variation of viscosity with temperature may produce short wavelength convection within regions heated by plumes. This question is important, since there is general agreement that the long wavelength topography of features such as Atla is maintained by convection.

There are, however, a number of difficulties in supporting short wavelength topography with a value of  $Z^f$  greater than 100 mGal/km by convection. The first is that the top of the convective circulation would have to be considerably deeper than 180 km, since a lithosphere of this thickness gives the value of  $Z^f$  of about 50 mGal/km seen at long wavelengths in Atla Regio. It is hard to understand how stresses associated with such deep convection could deform the surface of the planet. Short wavelength circulation is believed to be occurring on Earth beneath the oceanic lithosphere, but any associated gravity anomalies are much smaller than those maintained elastically. Their amplitude does not exceed  $\pm 10$  mGal and may well be smaller. Similar small scale circulation is less likely to occur on Venus, where the mantle viscosity is greater than on Earth.

Another reason elastic forces are likely to be responsible for the short wavelength topography and gravity is the similarity of the observed behavior of  $Z_o^f(k)$  on Venus to that on Earth, where the importance of elastic forces is not disputed. Plots for two such regions, Hawaii and Siberia, are illustrated in Fig. 6. At long wavelengths  $Z^f(k)$  for Hawaii is dominated by convective signals, but the short wavelength behavior is the same as that of other oceanic features, such as the Emperor Seamounts, that are not associated with active plumes (Watts and Cochran 1974, Watts 1978). The short wavelength value of  $Z^f$  is smaller for Hawaii than it is for eastern Siberia and Venus because the density of seawater reduces the density contrast associated with uncompensated topography at all wavelengths.

The discussion above and Fig. 14 show that the methods



FIG. 16. Solid dots and vertical lines are estimates and error bars, respectively, of  $Z^{f}(k)$  for the six regions in Fig. 15. Bold curve shows the best fitting flexural model corresponding to the minimum of misfit,  $H^{f}(T_{e})$ . Second diagram in each pair is the misfit  $H^{f}$  as a function of  $T_{e}$ .



FIG. 16—Continued

used to stabilize downward continuation distort  $Z_o^f(k)$  and make reliable values of  $T_e$  difficult to estimate by the methods that have been used previously. In contrast, the signal processing approach developed here uses linear methods to extract the topographic signal from the noise. It is free from inversion artifacts and provides estimates of  $Z_o^f$  and its standard deviation. The resulting expressions could also be used to estimate the gravity field at wavelengths shorter than 500 km from the observed topography.

# 6. REGIONAL VARIATIONS IN $T_e$

A less detailed study has been carried out of six equatorial regions of Venus using data from cycle 4, to discover whether regional variations in  $T_e$  occur. The boundaries of the boxes are shown in Fig. 15. All LOS gravity obtained at spacecraft elevations of less than 400 km was used to obtain two-dimensional grids in Mercator projection, and the grids windowed with three multitaper windows in each dimension to estimate  $Z^f$ . The box used for Atla Regio is slightly larger than that in Fig. 5 and gives a slightly larger value of  $T_e$ . The values of  $T_e$  and misfits are listed in the legend to Fig. 16, which shows the estimates and fits. In general the fit of the flexural model in the wavelength band used, 150–500 km, is satisfactory. In several cases the convective signal at wavelengths longer than 500 km is clear, and there is a smooth increase, from convective values of  $Z^f$  of about 50 mGal/km, to larger values of 100 mGal/km typical of elastic flexure. The plots of  $Z^f$  shown in Fig. 16 resemble similar plots for Earth shown in Figs. 6d and 6e.

The exception to these remarks are Figs. 16i and 16j for Aphrodite. The estimates of  $Z^{f}$  are all smaller than expected for elastic support, and their values do not increase at short wavelengths. The most likely cause of this behavior is that the topography used to calculate the LOS gravity is not noise-free, as the calculations assume. The effect of topographic noise is to reduce the value  $Z_{o}^{f}$  (Mc-Kenzie 1994). Radar returns from regions as rough as Aphrodite are not easily used to measure the planetary radius. Figure 17 shows returns from three adjacent footprints, 290-292, from orbit 1188 where it crosses the eastern part of Aphrodite. The energy in the returns is spread by the rough topography within the broad region illuminated by the altimeter. It is therefore difficult to pick the peak corresponding to the specular return. The worst case is 291, where there is a difference of 850 m in the radius



FIG. 17. Altimeter returns for four footprints of orbit 1188 before and after Doppler sharpening. The footprint numbers are given at the top left of each plot. The ellapsed time is in units of 200 psec, starting from an arbitrary time. The shaded regions show the templates used to determine the planetary radius for each time series, and the value obtained in kilometers. The first three, 290–292, are from an area of tessera in eastern Aphrodite, with the center of 291 being at  $-12.64^{\circ}N$ , 133.74°E. The last, 16, from a plains region to the north of Aphrodite at 8.38°N, 132.04°E. The values of the fits are taken from the ARCDRs.

picked by the Doppler sharpened and unsharpened returns, both of which are noisy. Signals like these are typical of those from tesserae, and can produce large errors in the estimated radius. As Fig. 17 shows, such problems cannot be identified by using the tabulated goodness of fit. This measure is lower for footprint 16, from the plains north of Aphrodite, where the return is very sharp, than it is for footprint 291. The two regions on either side of Aphrodite are best fit by values of  $T_e$  that are smaller than those of 30–35 km found elsewhere, though the goodness of fit  $G^f$ for Eistla is poor. The range of values that fit the LOS gravity is the same as that of 12–34 km obtained by Johnson





**FIG. 18.** Comparison of the lithospheric structure of Venus with that proposed by McKenzie (1989) for the Kaapvaal Craton of South Africa (b) and old oceanic lithosphere (c) by McKenzie and Bickle (1988). The bold line is the geotherm; the thin line is the solidus. The cross-hatched region represents the likely elastic thickness of the lithosphere from this study of Venus and from McKenzie and Fairhead (1997) for the terrestrial examples. The values of the parameters used are  $\eta = 3 \times 10^{20} (10^{19})$  Pa sec, surface temperature 450(0)°C, effective elastic thickness 30 (20 shields, 30

and Sandwell (1994) from the topography of coronae. On Earth gravity anomaly profiles across thrust fronts also yield estimates of  $T_e$  that are similar to those obtained by fitting  $Z_o^f(k)$  for the same regions (McKenzie and Fairhead 1997).

#### 7. DISCUSSION AND CONCLUSIONS

The most important result of this study is the demonstration that reliable estimates of the admittance between gravity and topography can be obtained from the LOS velocity of a spacecraft when the signal to noise ratio is 1/10 or less, and the coherence between the gravity calculated from the topography and that from the LOS velocity is no more than 1%. Application of the method to Magellan data shows that the value of  $T_e$  is about 30 km for several regions, and some evidence for smaller values near Aphrodite. The values agree well with those obtained previously from the topography of coronae (Sandwell and Schubert 1992, Johnson and Sandwell 1994). Estimates of the crustal density and thickness are not well constrained by the admittance estimates. The magnitude of the velocity signal used to estimate  $T_e$  is only about 1 mm/sec.

Figure 18 compares the thermal and mechanical structure of the surface regions of Venus with those of Archaean shields and old ocean basins on Earth. The interior potential temperature is estimated to be 1300°C, from the FeO concentrations measured by the Soviet landers (McKenzie et al. 1992). A higher upper mantle viscosity is required on Venus than on Earth to maintain the larger amplitude topographic and gravity anomalies associated with plumes. The thickness of the mechanical and thermal boundary layers is principally constrained by the need to generate modest quantities of melt (Nimmo and McKenzie, 1996). The resulting thermal structure of the mechanical boundary layer, combined with an estimate of 30 km for  $T_{\rm e}$ , requires elastic stresses to be supported up to temperatures of about 650°C for geological times. This value is slightly above the upper bound of 600°C estimated by Watts (1994) for oceanic regions on Earth. Therefore there is a suggestion that the crust and upper mantle need to maintain stresses at somewhat higher temperatures on Venus than they do on Earth.

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oceans) km, mechanical boundary layer thickness 95 (165 shields, 107 oceans) km and interior potential temperature  $1300(1300)^{\circ}$ C. The heat flux calculated from the thermal structure is 21 (23 shields, 34 oceans) mW m<sup>-2</sup>. The quantities in brackets are for Earth, the others for Venus.

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